1.6 Square Roots

You will need
- grid paper
- a calculator

GOAL
Determine the square roots of perfect squares.

Learn about the Math

The floor mat in gymnastics is a square with an area of 144 m^2.

? How can you calculate the dimensions of the square mat?

A. Use the **formula** *Area* = *length* × *width* to explain why the product of the dimensions of the mat is 144 m^2.

B. In the **equation** ☐ × ☐ = 144, ☐ represents both the length and width of the mat. Fill in the boxes with the missing numbers.

C. Why is the area of the mat a **perfect square**?

D. The mat problem can be solved by calculating the **square root** of 144. Explain why.

E. What are the dimensions of the mat? Show how you got your answer.

Ravi said, "It's easy to calculate a square root. Enter a number like 121. Then press the square root key: [√]. The number in the display, 11, is the square root."

Sandra replied, "You can check that a number is the square root if you multiply it by itself. You should get the original number again."

perfect square
the product of a whole number multiplied by itself; for example, 81 is a perfect square because it is 9 × 9

square root
a number when multiplied by itself equals the original number; for example, the square root of 81 is represented as $\sqrt{81}$ and is equal to 9 because 9 × 9 or $9^2 = 81$

Communication Tip

The power 11^2 or "11 squared" represents the area of an 11-by-11 square while $\sqrt{121}$ or "square root of 121" represents the side length of a square with an area of 121.

Reflecting

1. Why are there no perfect squares between 144 and 169?

2. How did you know that the last digit of the whole-number dimensions of the floor mat must be a 2 or an 8? (*Hint*: Make a table of all the squares from 1 to 10, to see the last digits of greater squares.)

3. Suppose that your calculator does not have a square root key. How could you still use your calculator to determine $\sqrt{144}$?

Work with the Math

Example 1: Determining a square root by guessing and testing

The floor mat in rhythmic gymnastics is a square with an area of 169 m^2.
What are its whole-number dimensions?

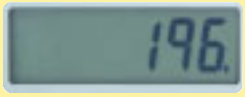
Yuki's Solution

$\square \times \square = 169$	I have to find two equal factors with a product of 169.
10×10 or $10^2 = 100$ Too low	The dimensions must be between 10 m and 20 m.
20×20 or $20^2 = 400$ Too high	
$3 \times 3 = 9$	I know that the last digit of the dimensions must be 3, because 9 is the last digit of 169.
13×13 or $13^2 = 169$	The dimensions of the square must be 13 m by 13 m.
$\sqrt{169} = 13$	This means that the square root of 169 must be 13.

Example 2: Using the square root key on a calculator

In artistic gymnastics, the square floor mat has an area of 196 m^2.
What are its whole-number dimensions?

Ryan's Solution

196 [√]	I entered 196, the area of the square mat, into my calculator.
	Then I pressed the [√] key to calculate the square root.
	The dimensions of the square must be 14 m by 14 m.
[×] [=]	I checked my answer by squaring 14.
	$14^2 = 14 \times 14$ $= 196$

A Checking

4. The "tatami," or mats, in judo are squares with a minimum area of 36 m^2 and a maximum area of 64 m^2.

a) Sketch diagrams of the mats on grid paper.

b) What are the possible whole-number dimensions of the mats? Check by multiplying.

5. Use mental math to calculate.

a) $\sqrt{4}$ **b)** $\sqrt{16}$ **c)** $\sqrt{81}$

6. When 32 is multiplied by itself, the product is 1024. What is the square root of 1024?

7. The square root of a number is 11. What is the number?

B Practising

8. The area of a square weightlifting platform is 16 m^2.

a) Sketch the platform. What are its dimensions?

b) What is the perimeter of the platform?

9. a) Explain how you know that the square root of 225 is between 10 and 20.

b) Will the square root of 225 be closer to 10 or 20? Explain.

c) Guess and test to find the square root of each number.

i) 289 **iii)** 2209 **v)** 8649
ii) 3025 **iv)** 3721

10. Use mental math to determine the square root of each number.

a) 1 **d)** 100
b) 0 **e)** 400
c) 25 **f)** 900

11. The preferred overall competition area in judo, including the mats, is a square with an area of 256 m^2.

a) How do you know that the length and width of the competition area are between 10 m and 20 m?

b) What are the possible last digits of the dimensions if the side lengths are whole numbers?

c) Use your answers to parts (a) and (b) to predict the dimensions.

d) Check your prediction using a calculator.

12. The number 121 is the first perfect square that is greater than 100. Calculate the following numbers. Show the steps you used to calculate each answer.

a) all perfect squares between 121 and 200

b) the first perfect square greater than 1000

13. Explain two different ways to calculate the square root of 225.

14. a) Explain how you know that $\sqrt{441}$ must be close to 20.

b) Explain how you know that the last digit of $\sqrt{441}$ must be 1 or 9 if the answer is a whole number.

c) Use your answers to parts (a) and (b) to predict $\sqrt{441}$.

d) Use a calculator to check your prediction.

C Extending

15. Explain how you can calculate each square root mentally.

a) $\sqrt{31 \times 31}$

b) $\sqrt{431 \times 431}$

c) $\sqrt{17^2}$

d) $\sqrt{43^2}$

16. Calculate the dimensions of a square that has the same area as a 16 m by 64 m rectangle. Show your steps.

17. Calculate the area of your classroom floor. Estimate the dimensions of a square that has the same area as your classroom floor. Check your estimate.

18. a) Calculate the square root of each power of 10.

i) $\sqrt{100}$

ii) $\sqrt{10\ 000}$

iii) $\sqrt{1\ 000\ 000}$

b) Describe the pattern for the number of zeros in each square root.

c) Use your pattern from part (b) to predict the number of zeros in $\sqrt{100\ 000\ 000}$.

19. Why might squaring a number and taking the square root of a number be thought of as opposite operations? Give an example to justify your answer.

Math Game

ROLLING POWERS

Number of players: 2 to 4

Rules

1. Roll one die to get a number that will be the base. Roll the other die to get the exponent.
2. Before the second roll, each player predicts whether the power will be greater than or less than 100.
3. The players calculate the answer.
4. Players score 1 point for each correct prediction.
5. Take turns rolling the dice.
6. The first player to reach 10 points wins.

You will need

- 2 dice
- a calculator
- paper and pencil