

5.5

Calculating Area

- You will need**
- a calculator
 - a compass
 - scissors

▶ GOAL

Develop and apply the formula for the area of a circle.

Learn about the Math

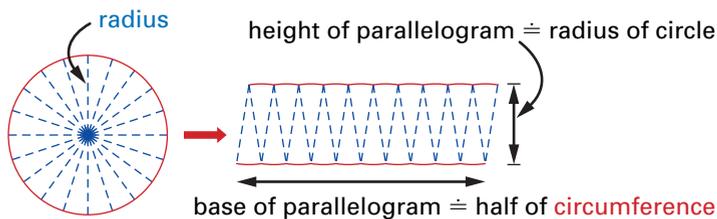
Teo is arranging water sprinklers. Each sprinkler sprays water in a circle with a diameter of 8.0 m.



? What area of grass does each sprinkler water?

Teo uses the formula for the area of a parallelogram to develop a formula for the area of a circle. He draws a circle with a diameter of 8.0 cm to model the area that is watered.

He cuts the circle into 20 equal sections. Then he arranges the sections into a parallelogram.



Area of parallelogram = base \times height

Area of parallelogram $\doteq \frac{1}{2}$ of circumference \times radius

Area of circle = $\frac{1}{2}$ of circumference \times radius

The height of the parallelogram is close to the radius of the circle. The base of the parallelogram is about half the circumference of the circle. So, the area of the parallelogram can be used to calculate the area of the circle, using the radius and circumference of the circle.

The formula for the area, A , of a circle with circumference C and radius r is

$$\begin{aligned}
 A &= \frac{1}{2} \times C \times r && \text{Use } C = 2\pi r. \\
 &= \frac{1}{2} \times 2\pi r \times r \\
 &= 2 \times \frac{1}{2} \times \pi \times r \times r \\
 &= 1 \times \pi \times r^2 \\
 &= \pi r^2
 \end{aligned}$$

Example 1: Calculating the area of a circle

A sprinkler waters in a circle with a diameter of 8.0 m. Determine the area that is watered by the sprinkler.

Teo's Solution: Using a formula

$$\begin{aligned} A &= \pi r^2 \\ &\doteq 3.14 \times (4.0 \text{ m})^2 \\ &\doteq 50.2 \text{ m}^2 \end{aligned}$$

The area that is watered is about 50.2 m².

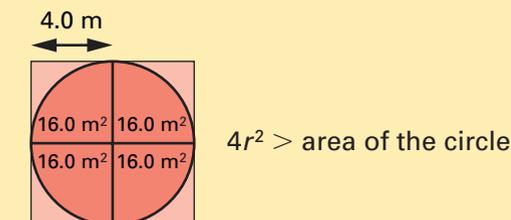
I used the formula for the area of a circle. The diameter of the circular area is 8.0 m, so the radius is 4.0 m.

My answer uses the same number of decimal places as is given in the problem.

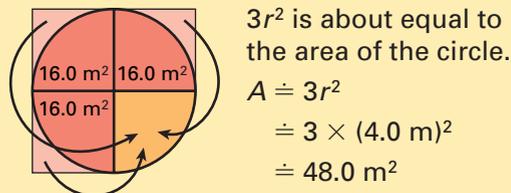


Manuel's Solution: Using the areas of squares

I drew a circle to model the watered area. I covered the circle with paper squares. The sides of the squares are the same length as the radius of the circle.



Try four squares. The area of one square is r^2 , so the area of four squares is $4r^2$, or 64.0 m². Four squares definitely cover more area than the circle does.



Try three squares. The area of three squares is $3r^2$, or 48.0 m². Three squares seem to cover almost the same area that the circle does. Altogether, the parts of the squares that are outside the circle seem to be about the same area as the uncovered part of the circle.

The area that is watered is a little more than 48.0 m².

Since πr^2 is $3.14r^2$, it is reasonable that the area of the circle should be a little more than $3r^2$.



Reflecting

1. How does knowing the formula for the circumference of a circle help you develop the formula for the area of a circle?
2. How can you calculate the area of a circle if you know its radius? What if you know its diameter?
3. Why would you use a formula to determine the area of a circle?

Work with the Math

Example 2: Calculating the area of a circle

A metal cover is needed for a circular fire pit with a radius of 250 cm. What is the area of the fire pit?

Solution

Substitute the radius in the formula for the area of a circle.

$$\begin{aligned} A &= \pi r^2 \\ &\approx 3.14 \times (250 \text{ cm})^2 \\ &\approx 196\,250 \text{ cm}^2 \end{aligned}$$

The answer is a lot of square centimetres, so I'd rather use square metres. The area of the fire pit is about 19.63 m².

Round your answers to the same number of decimal places as in the given measurements.

A Checking

4. Determine the area of each object. Use the formula $A = \pi r^2$.

- a) radius 10.5 cm



- b) diameter 14 cm



- c) radius 13 cm



- d) diameter 2.8 cm

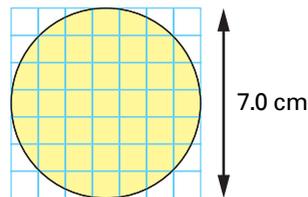


B Practising

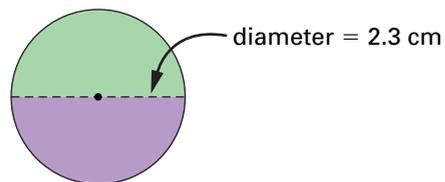
5. Calculate the area of a circle with each measurement.

- diameter 7.3 cm
- radius 2 cm
- radius 2.7 cm
- diameter 1.7 cm

6. Explain how to estimate and then calculate the area of the circle.

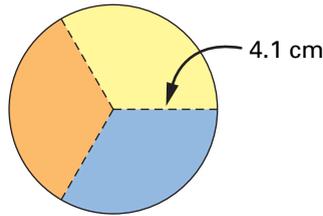


- What area does this circle cover?
- What is the area of each section?



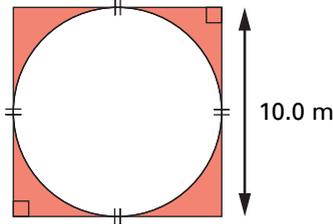
8. a) What area does this circle cover?
 b) The three sections are equal.

What is the area of each section?



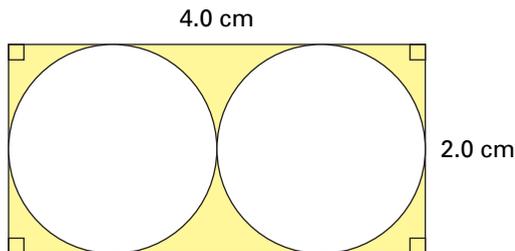
9. The radius of a circular pizza is 22.0 cm.
 a) What area does the pizza cover?
 b) If the pizza is cut into four equal pieces, what is the area of each piece?

10. a) What is the area of the square?
 b) What is the area of the white circle?
 c) What is the total area of the four red sections?



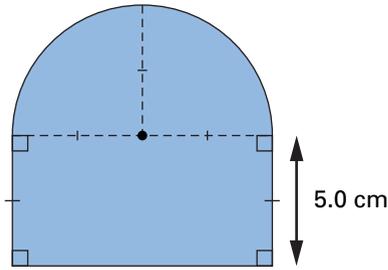
11. Explain the steps you would take to calculate the area of a circle with a circumference of 10.0 cm.

12. a) What is the area of the rectangle?
 b) What is the total area of the six yellow sections?



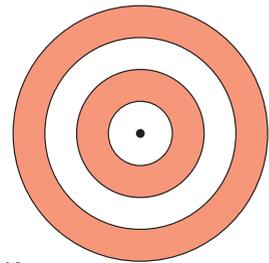
13. The diameter of a DVD is 12.0 cm. The diameter of the hole in the centre is 1.5 cm. Use this information to write a problem that involves area. Then solve your problem.

14. What is the total area of this figure?

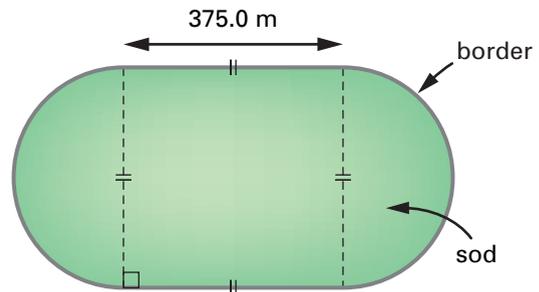


C Extending

15. Carla drew this target on the floor for a bean bag game. The radius of the target is 30.0 cm, and all bands are the same width. What area of the target is red?



16. Roberto has designed this park for a new housing development. The park is a square with a semicircle at each end. It will be covered with sod and have a border made of paving stones.



- a) What area of sod is needed to cover the park?
 b) Sod costs \$1.25/m². How much will the sod for the park cost?
 c) How long will the border be?
 d) The paving stones cost \$2.75/m. How much will the border around the park cost?
 e) How much will the sod and paving stones cost, in total?