Solve Problems by Working Backward

▶ **GOAL**
Work backward to solve problems.

Learn about the Math

John uses a trundle wheel to measure distance. Each time the wheel makes one complete turn of one metre, it clicks.

? How can you draw a full-sized diagram of a trundle wheel?

1 Understand the Problem

Manuel is going to use a compass to draw a diagram of the trundle wheel. The circumference of the trundle wheel is 1 m. Manuel needs to know the radius.

2 Make a Plan

Manuel knows the formula for circumference. He can work backward from the formula to determine the radius.

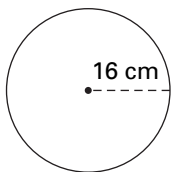**3 Carry Out the Plan**

$C = \pi d$ Start with the formula for circumference.

$1.00 \text{ m} = \pi d$ Substitute the value of the circumference.

$1.00 \text{ m} \div \pi = d$ Calculate the diameter in metres.

$1.00 \text{ m} \div 3.14 \doteq d$

$0.32 \text{ m} \doteq d$ The diameter is about 0.32 m, or 32 cm.

The radius is half the diameter, so the radius is about 16 cm.

16 cm

4 Look Back

Manuel drew a circle with a radius of 16 cm. He measured the circumference with string to check that it was about 1 m.

Reflecting

1. What relationships about circle measurements were needed to solve the problem?

2. How did Manuel work backward to solve the problem?

Work with the Math

Example 1: Constructing a circle with a known area

A pizza parlour sells circular pizzas and square pizzas that require the same amount of dough. The box for a square pizza is 40 cm by 40 cm. What is the area of the smallest box that a circular pizza will fit into?

square pizza   40 cm
40 cm

round pizza   radius of pizza

side length of box = diameter of pizza
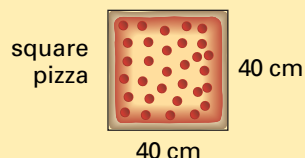
Reilly's Solution

1 Understand the Problem

The circular pizza requires the same amount of dough as the square pizza.

2 Make a Plan

I can calculate the area of the square pizza from the side length. I know that the formula for the area of a circle includes the radius. I can use this formula to work backward to determine the radius of the round pizza.

3 Carry Out the Plan

$$A = \pi r^2$$ I start with the area formula.

$$1600 \text{ cm}^2 = \pi r^2$$ The area of the square pizza is $40 \text{ cm} \times 40 \text{ cm} = 1600 \text{ cm}^2$.

$$1600 \text{ cm}^2 \div \pi = r^2$$

$$510 \text{ cm}^2 \doteq r^2$$ The radius must be the square root of 510.

$$23 \text{ cm} \doteq r$$

The radius is about 23 cm, so the diameter is about 46 cm (to the nearest centimetre). The box is 46 cm by 46 cm.
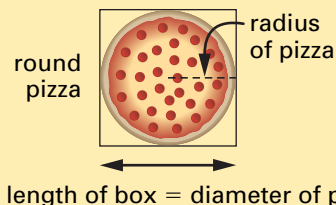
4 Look Back

I'll check by substituting the value of the radius into the area formula.

$$A = \pi r^2$$
$$\doteq 3.14 \times (23 \text{ cm})^2$$
$$\doteq 1661 \text{ cm}^2$$

There is a slight difference in the answers due to rounding. I think my answer is correct.

Example 2: Calculating the original price

During a clothing sale, the price of an item goes down by half each day that it is not sold. If a jacket costs $4.50 after six days, what was the original price?

Tamara's Solution
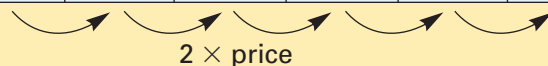
1 Understand the Problem

I know that the jacket costs $4.50 now. I want to know what it cost six days ago.

2 Make a Plan

The price was halved each day. Doubling is the opposite of halving. I will start at the price on day 6 and double the price back to day 1. This will give me the original price.

3 Carry Out the Plan

Day	6	5	4	3	2	1
Price ($)	4.50	9.00	18.00	36.00	72.00	144.00

2 × price

The original price of the jacket was $144.00.

4 Look Back

To check my work, I start with $144.00 and take half each day to day 6. I end up with $4.50, so my answer is correct.
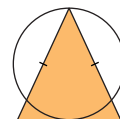
A Checking

3. Draw a circle with each measurement.

   a) circumference 62.8 cm

   **b)** area 314 cm^2

B Practising

4. Write the calculations in order, to determine the original number.
   - Choose a number.
   - Add −12.
   - Subtract −10.
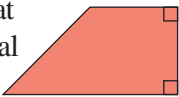   - Use its opposite.
   - Add 12.
   - The answer is −32.

5. The area of the circle in this figure is 78.5 m^2. What is the height of the triangle?

6. This figure covers an area of 706 cm^2.

   a) What is the radius of each circle?

   b) What is the circumference of each circle?

   c) What area is covered by each colour?
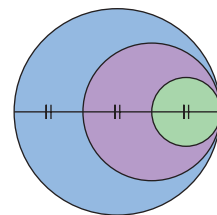
7. Fatouma is lifting weights over a 10-week training period. Every week, she lifts 2 kg more than she lifted the previous week. During the tenth week, she lifts 120 kg. What mass did she lift during the first week?

8. Henri cut away half of a shape five times. The following trapezoid is what remains. Draw what the original shape might have been.

9. Eastward High School needs a new logo for the gym floor. The logo must fit inside a square with an area of 25.0 m² and be bordered by a circle.

   a) A line for basketball tip-offs will run through the middle of the circle. How long will this line be?

   b) What will be the circumference of the circle around the logo?

10. The rim of a bicycle wheel has a circumference of 256.3 cm. Use this information to write three problems that can be solved by working backward. Solve your problems.

11. A lawn sprinkler rotates and sprays water in a circle. It sprays an area of 283 m². How far can it spray?

12. The Canadian $2 coin has a copper centre surrounded by a ring of nickel. The diameter of the coin is 2.8 cm. The area of the copper centre is 2.0 cm². What is the area of the nickel ring?

Curious Math

CUTTING PAPER STRIPS

Did you know that a piece of paper can have just one side?

   1. Give a strip of paper half a twist. Tape the ends together.

   2. Draw a line down the middle of the paper band. What do you think will happen if you cut along the line? Try it.

You will need
- strips of paper
- tape
- scissors

   3. Tape two circular strips together as shown. Cut around the circumference of the two strips. Can you make a rectangle or a parallelogram?