

11.2

Surface Area of a Cylinder

You will need

- a calculator
- centimetre grid paper
- a ruler
- a compass

▶ GOAL

Develop and apply a formula for calculating the surface area of a cylinder.

Learn about the Math

Toma and Maria are wrapping tea lights to sell for a school fundraiser. They are wrapping the tea lights in stacks of five.



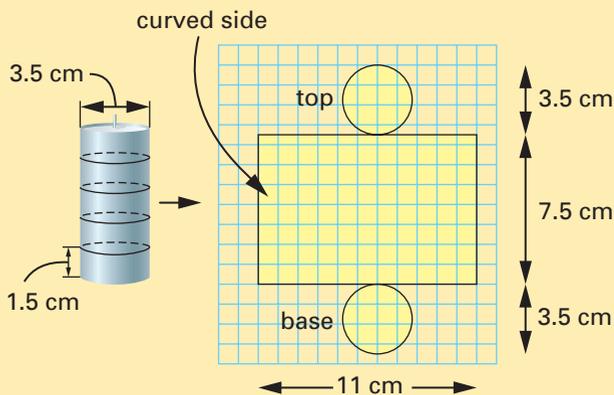
? How much paper do Toma and Maria need to wrap each stack of tea lights?

Example 1: Estimating surface area with grid paper

Use a net to estimate the amount of wrapping paper that Toma and Maria need to wrap each stack of five tea lights.

Toma's Solution

I imagined unwrapping the package and laying the paper flat. I traced the circular base and top of the cylinder on grid paper.



The height of each tea light is 1.5 cm, so the height of the curved side is $5 \times 1.5 \text{ cm} = 7.5 \text{ cm}$.

The width of the curved side is the same as the circumference of the base. The diameter of the base is 3.5 cm, so the circumference is $3.5 \text{ cm} \times \pi \doteq 11 \text{ cm}$.

I counted the squares in the net. There are about 10 squares in each circle. There are about $7.5 \times 11 \doteq 83$ squares in the curved side (rectangle). Each square has an area of 1 cm^2 .

$$\begin{aligned} \text{Total surface area} &= \text{area of base} + \text{area of top} + \text{area of curved side} \\ &\doteq 10 \text{ cm}^2 + 10 \text{ cm}^2 + 83 \text{ cm}^2 \\ &\doteq 103 \text{ cm}^2 \end{aligned}$$

Toma and Maria need about 103 cm^2 of wrapping paper for each stack of tea lights.



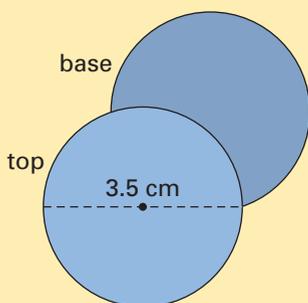
Example 2: Calculating surface area using a formula

Calculate the amount of wrapping paper that Toma and Maria need to wrap each stack of five tea lights.

Maria's Solution

The base and top of each stack are congruent faces, so they have the same area. I just need to determine the area of one face, and then I can double it.

The diameter is 3.5 cm, so the radius is $3.5 \text{ cm} \div 2 \doteq 1.8 \text{ cm}$.

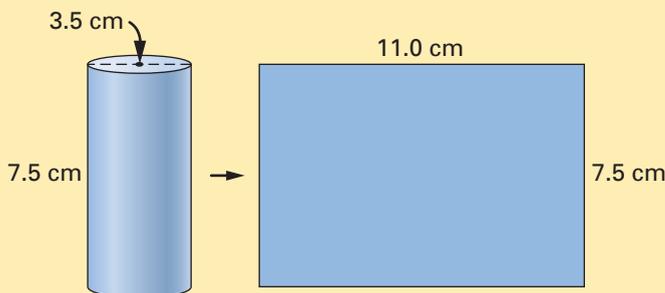


$$\begin{aligned}\text{Area of top and base} &= 2 \times \pi r^2 \\ &\doteq 2 \times \pi \times (1.8 \text{ cm})^2\end{aligned}$$

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2 * pi * 1.8 ^ 2 =  
20.3575204
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The area of the top and base is 20.4 cm^2 . I rounded to one decimal place because this is how the radius is given in the problem.

If I laid out the curved side of the package, it would form a rectangle. Its width would be equal to the circumference of the base. Its length would be equal to the height of the package. So, its area is equal to the circumference of the base multiplied by the height of the package.



$$\begin{aligned}\text{Area of curved surface} &= \text{circumference of base} \times \text{height} \\ &= \pi d \times \text{height} \\ &= (\pi \times 3.5 \text{ cm}) \times 7.5 \text{ cm}\end{aligned}$$

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pi * 3.5 * 7.5 =  
82.46680716
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The area of the curved surface is 82.5 cm^2 .

$$\begin{aligned}\text{Total surface area} &= \text{area of top and base} + \text{area of curved surface} \\ &\doteq 20.4 \text{ cm}^2 + 82.5 \text{ cm}^2 \\ &\doteq 102.9 \text{ cm}^2\end{aligned}$$

Toma and Maria need 102.9 cm^2 of wrapping paper for each stack of tea lights.



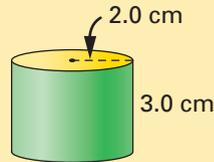
Reflecting

1. What part of the net for a cylinder is affected by the height of the cylinder?
2. What part of the net for a cylinder is affected by the size of the base?
3. Write a formula to calculate the surface area of a cylinder.

Work with the Math

Example 3: Calculating surface area using a formula

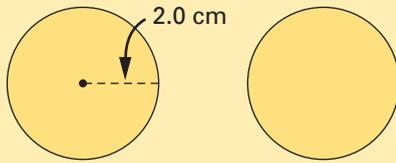
Calculate the surface area of this cylinder.



Solution

Surface area of cylinder = area of base and top + area of curved surface

Sketch the faces, and calculate the area of each face.



$$\begin{aligned} \text{Area of base and top} &= 2 \times \text{area of base} \\ &= 2 \times \pi r^2 \\ &\doteq 2 \times 3.14 \times (2.0 \text{ cm})^2 \\ &\doteq 25.1 \text{ cm}^2 \end{aligned}$$



$$\begin{aligned} \text{Width of rectangle} &= \text{circumference of circle} \\ &= 2\pi r \\ &\doteq 2 \times 3.14 \times 2.0 \text{ cm} \\ &\doteq 12.6 \text{ cm} \end{aligned}$$

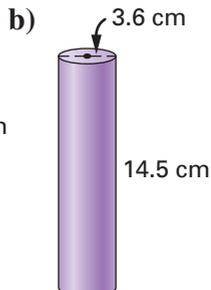
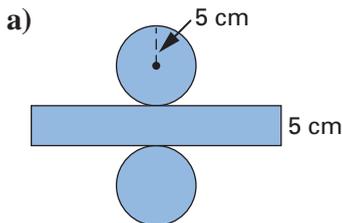
$$\begin{aligned} \text{Area of curved surface} &= \text{area of rectangle} \\ &= \text{length} \times \text{width} \\ &\doteq 3.0 \text{ cm} \times 12.6 \text{ cm} \\ &\doteq 37.8 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Surface area of cylinder} &= \text{area of base and top} + \text{area of curved surface} \\ &\doteq 25.1 \text{ cm}^2 + 37.8 \text{ cm}^2 \\ &\doteq 62.9 \text{ cm}^2 \end{aligned}$$

The surface area is 62.9 cm^2 .

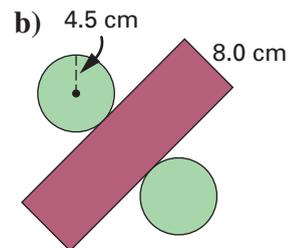
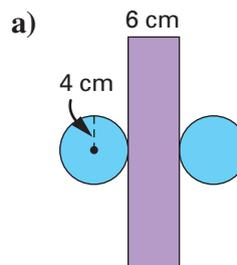
A Checking

4. Calculate the surface area of each cylinder.



B Practising

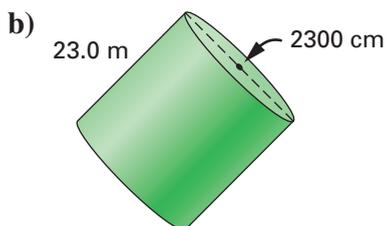
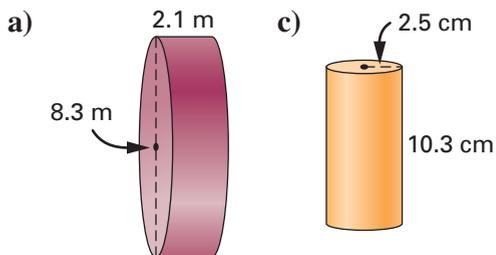
5. Use each net to determine the surface area of the cylinder.



6. Three cylinders have bases that are the same size. The area of the base is 10.0 cm^2 . Determine the surface area of each cylinder, given its height.

a) 8.0 cm b) 6.5 cm c) 9.4 cm

7. Calculate the surface area of each cylinder.



8. Describe how you would determine the surface area of a potato-chip container that is shaped like a cylinder.

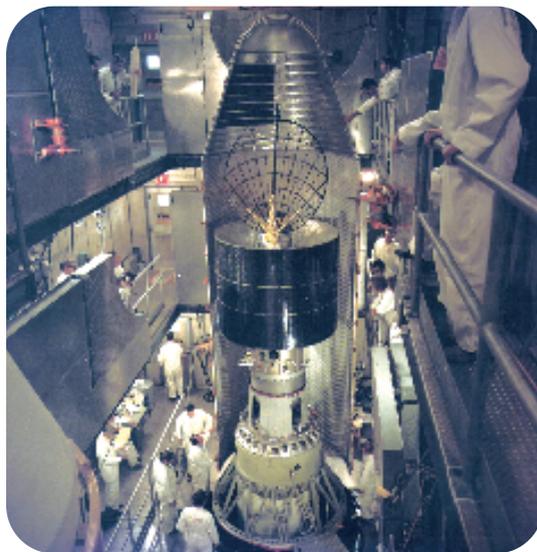
9. a) This railway car is 3.2 m in diameter and 17.2 m long. Calculate its surface area.



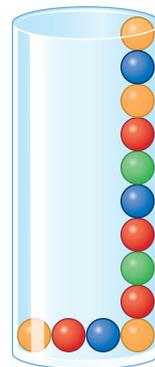
- b) Estimate the cost of painting the outside of the railway car, if a can of paint covers an area of 40 m^2 and costs \$35.

10. Explain why two cylinders that are the same height can have different surface areas.

11. “Anik” is the name of a series of Canadian communications satellites. The first Anik, shown here, was launched in 1972. It was a cylindrical shape, 3.5 m high and 190 cm in diameter. All satellites are wrapped with insulation because the instruments inside will not work if they become too cold or hot. What was the approximate area of the insulation used to wrap the first Anik?



12. A games shop sells marbles in clear plastic cylinders. Four marbles fit across the diameter of the cylinder, and 10 marbles fit from the base to the top of the cylinder. Each marble has a diameter of 2 cm. What is the area of the plastic that is needed to make one cylinder?



C Extending

13. Gurjit has a CD case that is a cylindrical shape. It has a surface area of 603 cm^2 and a height of 10 cm. What is the area of the circular lid of the CD case?