Rates and Ratios

Curriculum Ready

Mathletics

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Rates and ratios are ways of comparing two or more quantities.

Rates and ratios are used all around us every day.

Recipes	Speed	Rate of Pay
		
The ratio of the different ingredients in a recipe.	The rate of distance travelled with respect to time in a car, km/h.	The amount a person earns per week can be expressed as a rate of dollars per hour. For example, $18 per hour.

Can you list some other examples in everyday life where two or more quantities are compared?

Q Harry and Sally purchased a bag containing 36 cookies.
They decide to share them based on how much each contributed to their purchase.
If Harry paid 3 times as much money as Sally did for this purchase, how many cookies does he keep?

Work through the book for a great way to do this

Ratios

A ratio compares two quantities in a given order.

For example, if the number of oranges in a bag is twice that of the number of apples, this is a ratio of 2 to 1 and is written as:

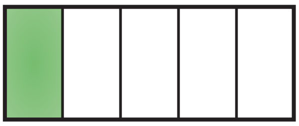
$$2:1$$

Each number is called a term of the ratio

The order in a ratio is important. The ratio of oranges to apples is $2:1$. The ratio of apples to oranges is $1:2$.

The shapes below are divided into equal rectangular regions. For each region:
(i) Write down the ratio of the shaded region to unshaded region.
(ii) Write down the ratio of the shaded region to the whole shape.

a



(i) 1 part is shaded, while 4 parts are unshaded, so the ratio of shaded to unshaded parts is 1 to 4, or using ratio notation, $1:4$.

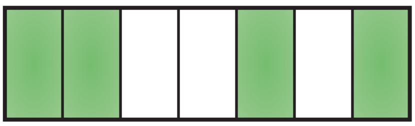
(ii) There are 5 equal parts in total.
$\therefore$ The ratio of shaded to the whole is $1:5$.

b

(i) 3 parts are shaded, while 2 parts are unshaded, so the ratio of shaded to unshaded parts is 3 to 2, or using ratio notation, $3:2$.

(ii) There are 5 equal parts in total.
$\therefore$ The ratio of the shaded parts to the whole shape is $3:5$.

c



(i) 4 parts are shaded, while 3 parts are unshaded.
$\therefore$ The ratio of shaded to unshaded parts is 4 to 3, or using ratio notation, $4:3$.

(ii) There are 7 equal parts in total.
$\therefore$ The ratio of the shaded parts to the whole shape is $4:7$.

Ratios

1 **a** If there are 17 girls and 12 boys in a classroom, then:

(i) The ratio of girls to boys is: ☐ : ☐.

(ii) The ratio of boys to girls is: ☐ : ☐.

(iii) The ratio of all students in the classroom to the number of boys is: ☐ : ☐.

(iv) The ratio of the number of girls in the classroom to the total number of students is:

☐ : ☐.

b A school has 41 basketball players, 23 soccer players, 31 netballers, 29 swimmers, 61 Rugby players, 17 tennis players, 12 lawn bowlers, and 8 who do not play sport. Find:

(i) The ratio of basketball players to rugby players: ☐ : ☐.

(ii) The ratio of soccer players to swimmers: ☐ : ☐.

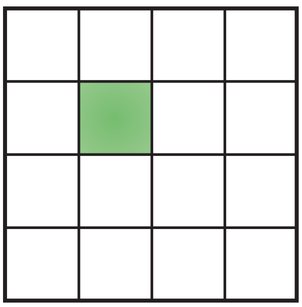
(iii) The ratio of rugby players to soccer players: ☐ : ☐.

(iv) The ratio of tennis players to netballers: ☐ : ☐.

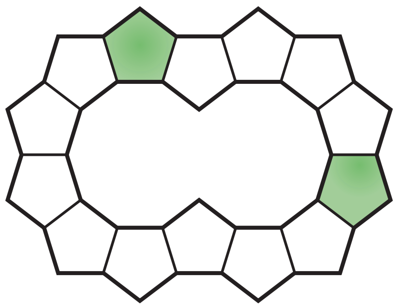
(v) The ratio of sport players to non-players: ☐ : ☐.

2 Complete shading in the parts until they match the ratio of `shaded to non-shaded´ given underneath.

psst. It doesn't matter which ones you shade, as long as the proportion is correct.



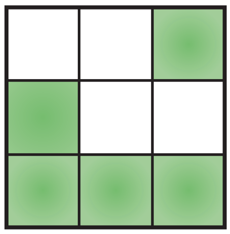
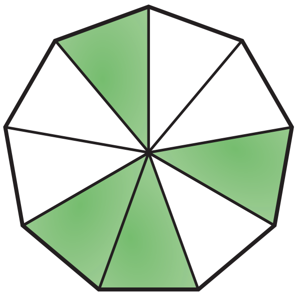
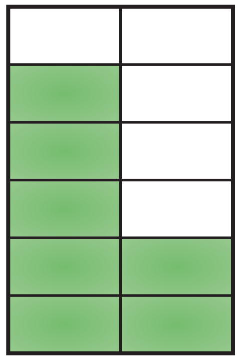
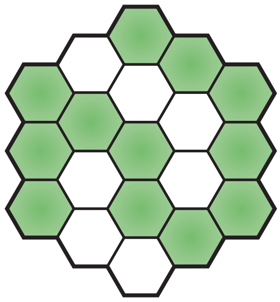
Shaded to Non-shaded parts $= 7:9$.



Shaded to Non-shaded parts $= 9:5$.

Ratios

3 (i) Write the ratios of the shaded regions to the unshaded regions for each diagram below.

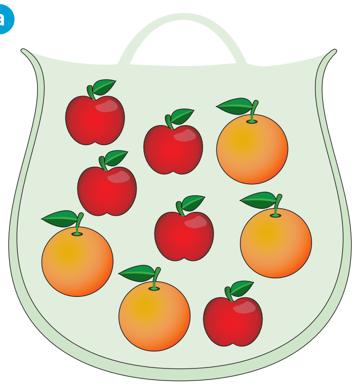
☐ : ☐ ☐ : ☐ ☐ : ☐ ☐ : ☐

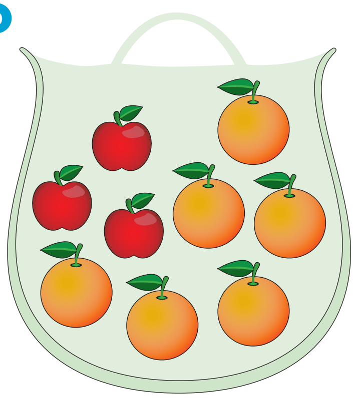
(ii) Write the ratios of the shaded parts to the total parts for each diagram.

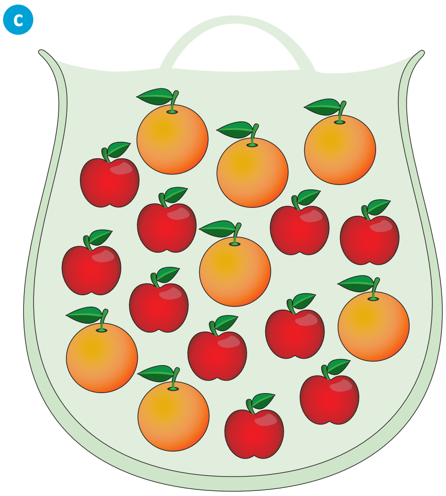
☐ : ☐ ☐ : ☐ ☐ : ☐ ☐ : ☐

4 (i) Write down the ratio of apples to oranges in each bag.

(ii) What is the ratio of oranges to the number of fruit in the bag?

a


b


c


a (i) ☐ : ☐ (ii) ☐ : ☐

b (i) ☐ : ☐ (ii) ☐ : ☐

c (i) ☐ : ☐ (ii) ☐ : ☐

Equivalent ratios

Ratios are equivalent if they represent the same relative proportions between two quantities.

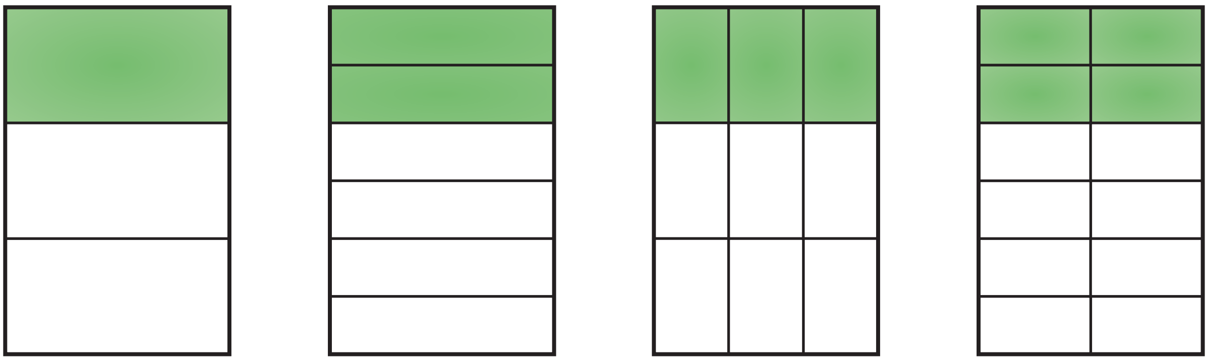
Using shading to demonstrate equivalent ratios

These rectangles are all the same size and divided into different numbers of equal parts.

The proportion of the shaded to unshaded areas are the same.

So the ratios of shaded to unshaded for each rectangle are equivalent.

$1:2$ $\quad$ $2:4$ $\quad$ $3:6$ $\quad$ $4:8$



Lowest terms	$2:4$	$3:6$	$4:8$
	$\div 2 \quad \div 2$	$\div 3 \quad \div 3$	$\div 2 \quad \div 2$
	$1:2$	$1:2$	$1:2$
	$\times 2 \quad \times 2$	$\times 3 \quad \times 3$	$\times 2 \quad \times 2$
	$2:4$	$3:6$	$4:8$

Equivalent ratios are found by multiplying or dividing all terms in the ratio by the same number.

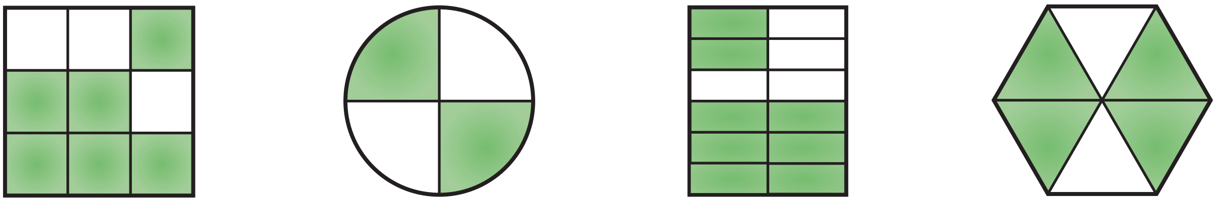
A ratio involving only rational numbers is said to be in **lowest terms** or in **simplest form**, if all terms are integers without any common factors.

- The ratio $2:4$ is not in lowest terms, as you can still divide both terms by 2 to obtain $1:2$.

- The ratio $1:2$ is in lowest terms and cannot be simplified further.

Equivalent ratios

1 (i) Write the ratios of the shaded to unshaded parts for these diagrams.

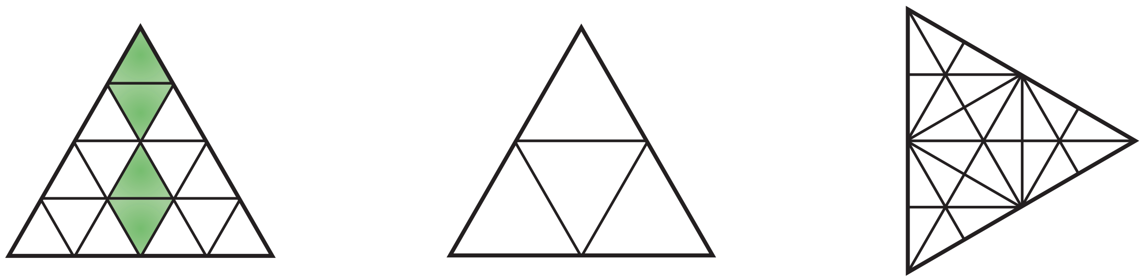


= 6 : 3   = ☐ : ☐   = ☐ : ☐   = ☐ : ☐

(ii) Write each of these ratios in lowest terms.

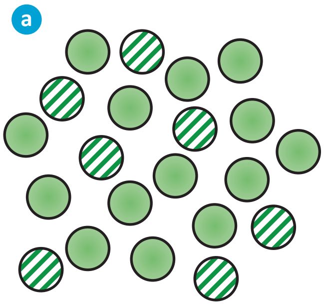
= 2 : 1   = ☐ : ☐   = ☐ : ☐   = ☐ : ☐

2 Shade the next two shapes to represent equivalent ratios of shaded to unshaded in the first shape.



3 (i) Write down the ratios of striped shapes to solid shapes, and reduce the ratio to simplest form.

a

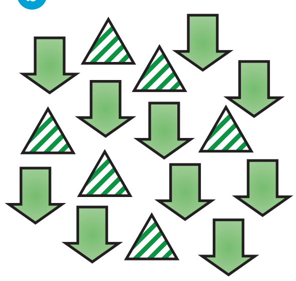


= ☐ : ☐

= ☐ : ☐

simplest form

b

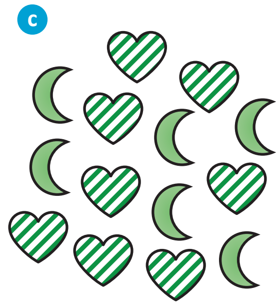


= ☐ : ☐

= ☐ : ☐

simplest form

c



= ☐ : ☐

= ☐ : ☐

simplest form

(ii) Write another ratio equivalent to each of the ratios above in (i).

= ☐ : ☐   = ☐ : ☐   = ☐ : ☐

Equivalent ratios

EQUIVALENT RATIOS * EQUIVALENT RATIOS * ?:? ..../..../20....

4 Simplify these ratios to lowest terms by dividing by greatest common factors.

a) 2 : 10

b) 10 : 30

c) 12 : 48

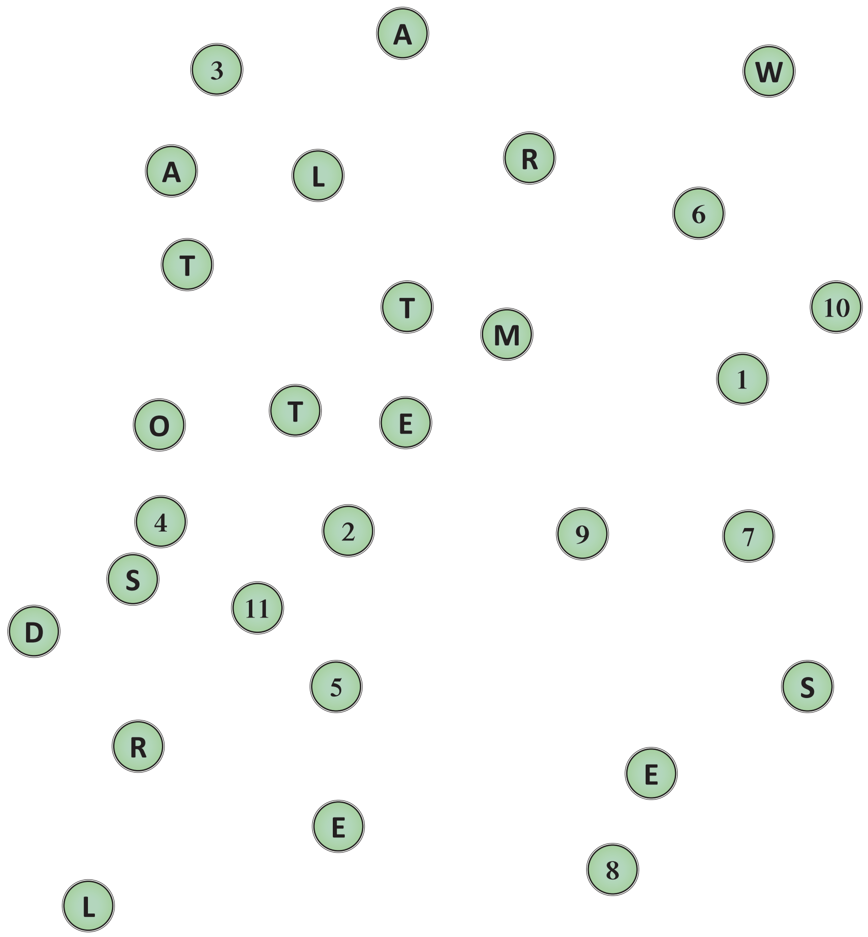
d) 14 : 35

e) 70 : 36

f) 18 : 81

5 Connect the equivalent ratios with a straight line to solve the puzzle.

1 : 2	3 : 5
1 : 3	23 : 69
8 : 3	19 : 4
21 : 35	18 : 4
64 : 16	9 : 180
1 : 20	1 : 1
95 : 20	5 : 10
85 : 75	6 : 5
90 : 20	24 : 9
45 : 51	15 : 17
12 : 9	17 : 15
123 : 123	4 : 1
180 : 150	48 : 36



1 2 3 4 5 6 7 8 9 10 11

Using fractions to reduce ratios in lowest terms

Simplifying ratios with whole numbers is similar to simplifying fractions. In fact we can write our ratios as fractions, reduce the fraction, and then convert back to ratio notation.

Examples of using fractions to simplify ratios problems

Reduce these to lowest terms by writing as fractions to simplify first.

a $4:32$

$$\frac{4}{32}=\frac{1}{8}$$ divide top and bottom by 4

The reduced form of the ratio is then $1:8$.

$$\therefore\ 4:32=1:8$$

You can also simplify quickly by using the fraction button on your calculator.

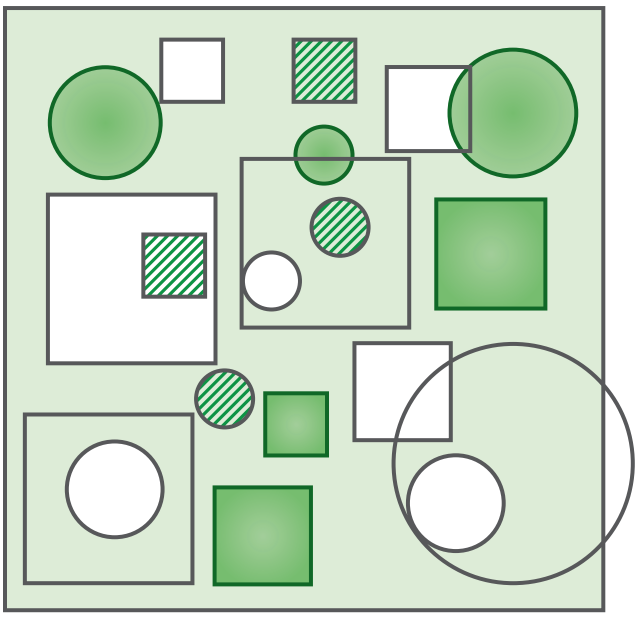
b $42:28$

$$\frac{42}{28}=\frac{3}{2}$$ divide top and bottom by 14

$$\therefore\ 42:28=3:2$$

Be careful using the calculator here as it might display $\frac{42}{28}$ simplified as $1\frac{1}{2}$ ($=\frac{3}{2}$)

c Ratio of circles to squares.



Ratio of circles to squares $=9:12$.

$$\frac{9}{12}=\frac{3}{4}$$

$$\therefore\ 9:12=3:4$$

Using fractions to reduce ratios in lowest terms

1 Reduce these ratios to lowest terms by converting to fractions, simplifying and back to ratio notation.

a $3:27$

$$\frac{3}{27} = \frac{\square}{\square}$$

$$\therefore\ 3:27 = \square : \square$$

b $12:30$

$$\frac{12}{30} = \frac{\square}{\square}$$

$$\therefore\ 12:30 = \square : \square$$

c $8:8$

$$\frac{8}{8} = \frac{\square}{\square}$$

$$\therefore\ 8:8 = \square : \square$$

d $20:100$

$$\frac{20}{100} = \frac{\square}{\square}$$

$$\therefore\ 20:100 = \square : \square$$

e $21:70$

$$\frac{21}{70} = \frac{\square}{\square}$$

$$\therefore\ 21:70 = \square : \square$$

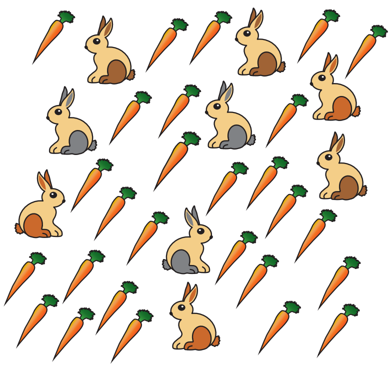
f $200:512$

$$\frac{200}{512} = \frac{\square}{\square}$$

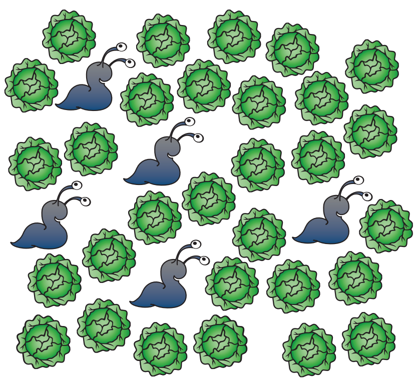
$$\therefore\ 200:512 = \square : \square$$

2 Use the fraction method to reduce each of these ratios to lowest terms.

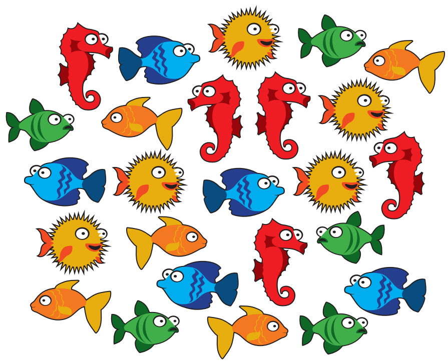
a Rabbits to carrots



b Lettuce to slugs



c Spikey puffer fish () to all fish



Ratios with decimals

If the terms of a ratio are terminating decimals, some simple rules allow easy simplification to lowest terms.

1. Move the decimal point in the ratio by the same number of places for all terms until you have only whole numbers on both sides.
2. Simplify the ratio to lowest form.

Simplify the ratios to lowest terms

a $2.2:5.4$

Both numbers have one decimal place, and so by moving the decimal by one place to the right will make both into integers (by multiplying both numbers in the ratio by 10).

$$2.2:5.4$$

$2.2:5.4$ — Move the decimal once to the right

$= 22:54$

$= \frac{22}{2}:\frac{54}{2}$ — Divide the ratio by 2 (both the left and right number of the ratio)

$= 11:27$

$\therefore\ 2.2:5.4 = 11:27$ ← This is now in lowest terms.

b $0.05:1.2$

The left decimal, 0.05 consists of two decimal places, and the number on the right, 1.2, has one decimal place.

Moving each decimal twice to the right will make both sides into integers.
This is equivalent to multiplying by 100.

$$0.05:1.20$$

$0.05:1.2$ — $\times 100$ on both sides of the ratio. Move the decimal by 2 places to the right.

$= 5:120$

$= \frac{5}{5}:\frac{120}{5}$ — Divide the ratio by 5 (both the left and right number of the ratio)

$= 1:24$

$\therefore\ 0.05:1.2 = 1:24$ ← This is now in lowest terms.

Ratios with decimals

1 Simplify these ratios by moving the decimal in both numbers one place to the right, and then reduce to lowest terms:

a) $1.5:4.5$

b) $1.2:7.2$

c) $9.2:2.4$

2 Simplify these ratios by moving the decimal in both numbers two places to the right, and then reduce to lowest terms:

a) $0.08:0.60$

b) $2.8:0.21$

c) $2:3.52$

3 Simplify these ratios by moving the decimal in both numbers by three places to the right, and then reduce to lowest terms using your calculator:

a) $0.008:0.018$

b) $0.025:0.100$

c) $2.40:1.84$

d) $6.42:0.60$

e) $1.24:4.48$

f) $0.022:0.40$

Ratios with fractions

For ratios containing fraction terms, we can use two methods to find the ratio in lowest integer terms.

1. Write their equivalent fraction with a common denominator, or
2. Cross multiply the denominators.

Examples of reducing ratios involving fractions to lowest form.

a $\frac{1}{2} : \frac{1}{3}$

$\frac{1}{2} : \frac{1}{3}$

$\frac{1}{2} \times \frac{3}{3} : \frac{1}{3} \times \frac{2}{2}$ — The lowest common multiple of 2 and 3 is 6 so this is a good common denominator.

$\frac{3}{\cancel{6}} : \frac{2}{\cancel{6}}$ — Cancel the common denominator 6.

$3:2$

Alternatively just cross multiply: $\frac{1}{2} : \frac{1}{3}$

$\therefore\ 1 \times 3 : 2 \times 1$

$\therefore\ 3:2$ as before.

b $2 : \frac{3}{5}$

$2 : \frac{3}{5}$

$\frac{2}{1} : \frac{3}{5}$

$\frac{2}{1} \times \frac{5}{5} : \frac{3}{5} \times \frac{1}{1}$ — The lowest common multiple of 1 and 5 is 5 so this is a good common denominator.

$\frac{10}{\cancel{5}} : \frac{3}{\cancel{5}}$ — Cancel the common denominator 5.

$10:3$

Alternatively just cross multiply: $\frac{2}{1} : \frac{3}{5}$

$\therefore\ 2 \times 5 : 1 \times 3$

$\therefore\ 10:3$ as before.

b $2\frac{1}{4} : 1\frac{1}{5}$

$2\frac{1}{4} : 1\frac{1}{5} = \frac{9}{4} : \frac{6}{4}$ — Convert the mixed fractions to improper fractions.

$= \frac{9 \times 9}{4 \times 5} : \frac{6 \times 4}{5 \times 4}$ — Make the denominator the same, in this case it is 20.

$= \frac{45}{\cancel{20}} : \frac{24}{\cancel{20}}$ — Cancel the common denominator.

$= 45:24$

Ratios with fractions

1 Reduce these ratios to lowest terms using the method indicated, either common denominator or cross multiply:

a) $\frac{1}{5} : \frac{4}{5} = \square : \square$

b) $\frac{1}{2} : \frac{1}{7} = \frac{\square}{14} : \frac{\square}{14}$
$= \square : \square$

c) $\frac{2}{7} : \frac{1}{14} = 2 \times 14 : \square \times \square$
$= \square : \square$
$= \square : \square$

d) $\frac{3}{16} : 1 = \frac{3}{16} : \frac{\square}{\square}$
$= \square \times \square : \square \times \square$
$= \square : \square$

2 Reduce these ratios to simplest form:

a) $\frac{4}{7} : \frac{2}{5}$

b) $\frac{2}{5} : 3\frac{1}{4}$

c) $2\frac{1}{5} : 2\frac{2}{3}$

d) $5\frac{1}{4} : 4\frac{2}{3}$

Best buys and the unitary method

Ratios can be used to decide which product is the best buy.

Examples using the unitary method

a Which is the best buy: $50\,\text{g}$ of cheese for $\$24$ or $70\,\text{g}$ of cheese for $\$35$?
Re-write each ratio so the left hand sides of the ratios match. Then compare the right hand sides.

$$50\,\text{g}: \$24 = 10\,\text{g}: \frac{\$24}{5}$$ Divided by 5.

$$= 10\,\text{g}: \$4.80$$

$$70\,\text{g}: \$35 = 10\,\text{g}: \frac{\$35}{7}$$ Divided by 7.

$$= 10\,\text{g}: \$5.00$$

The first cheese costs $\$4.80$ per $10\,\text{g}$, while the second one $\$5$ per $10\,\text{g}$. The first one is cheaper by $20\,\text{c}$.

The unitary method reduces one of the quantities to a '1' and then multiplies up to find the answer.

b If 9 mangos cost $\$40.50$, how much does each mango cost?

9 mangos costs $\$40.50$

$\therefore$ 1 mango costs $\$40.50 \div 9$

$\therefore$ each mango costs $\$4.50$

c What's the best buy, 12 pencils for $\$6.72$ or 17 pencils for $\$9.25$?
The method is to find the cost of one pencil and compare them.

12 pencils for $\$6.72$

$\therefore \frac{12}{12}$ pencils for $\frac{\$6.72}{12}$

$\therefore$ 1 pencil for $\$0.56$

17 pencils for $\$8.67$

$\therefore \frac{17}{17}$ pencils for $\frac{\$8.67}{17}$

$\therefore$ 1 pencil for $\$0.51$

Therefore the second one is slightly cheaper at 1 pencil for $\$0.51$.

d Caviar was $22\,\text{g}$ for $\$105$ at one store and $10\,\text{g}$ for $\$55$ at another store. Convert both of these to a cost per $100\,\text{g}$ to the nearest cent to find the best buy.

$22\,\text{g}$ for $\$105$

$\therefore 1g$ for $\$\frac{105}{22}$ or $\$105 \div 22$

$\therefore 1\,\text{g}$ for $\$4.7727$

$\therefore 100\,\text{g}$ for $\$477.27$

$10\,\text{g}$ for $\$55$

$\therefore 1\,\text{g}$ for $\$\frac{55}{10}$ or $\$105 \div 10$

$\therefore 1\,\text{g}$ for $\$5.50$

$\therefore 100\,\text{g}$ for $\$550.00$

The best buy is $22\,\text{g}$ for $\$105$.

Best buys and the unitary method

1 Find the cost per single item if:

a) 12 apples cost $\$6.60$ — 1 apple =

b) 150 g of cheese costs $\$4.50$ — 1 g cheese =

c) 7 avocados cost $\$15.75$ — 1 avocado =

d) 200 m of wire costs $\$356$ — 1 m wire =

2 Find the cost per single whole item if:

a) Half a packet of pegs cost $\$2.10$ — 1 packet of pegs =

b) $\frac{1}{4}$ watermelon cost $\$6.60$ — 1 watermelon =

c) 0.4 kg of pasta costs $\$1.55$ — 1 kg pasta =

d) 0.7 cm of gold costs $\$251$ — 1 cm of gold =

3 Which are the best buys?

a) 4 kg of fish for $\$24.00$ or
8 kg of fish for $\$50$?

[Hint: Multiply the first one by 2.]

b) 3 sacks of potatoes for $\$14$ or
4 sacks of potatoes for $\$17$?

[Hint: Multiply the first one by 4 and the second one by 3 to find what 12 sacks cost in each case.]

c) 5 sets of guitar strings for $\$78.75$ or
7 sets of guitar strings for $\$110.95$?

d) 8 loaves of sourdough bread for $\$41.60$ or
25 loaves of sourdough bread for $\$131.25$?

Best buys and the unitary method

4 Use the unitary method to decide.

a Who is faster?

- a man who can run 120 m in 13 seconds or
- a man who can run 220 m in 22 seconds.

[Hint: Find how far each can run in 1 second.]

b Which is more expensive to buy?

- 8 video games for $\$62.80$ or
- 13 video games for $\$100.23$

c Find the mode of transport that travels at the faster speed between two towns if:

- by bus you travel 162 km in 2.7 hours or
- by train you travel 215 km in 3.32 hours.

[Hint: Find out how much each mode travels in 1 hour.]

5 Find the cost 'per 100' for each of these items:

a 25 g of flour costs $\$0.20$ $\quad$ 1 g = ☐ $\quad$ $\therefore 100$ g = ☐

b 75 mL of lavender oil costs $\$7.65$ $\quad$ 1 mL = ☐ $\quad$ $\therefore 100$ mL = ☐

c 2000 mL of milk costs $\$4.85$ $\quad$ 1 mL = ☐ $\quad$ $\therefore 100$ mL = ☐

d $10\,800$ minutes of machine hire costs $\$175.00$ $\quad$ 1 mL = ☐ $\quad$ $\therefore 100$ mL = ☐

Best buys and the unitary method

6 Find the cost per 100 g to decide on the best buy between:

a 54 g of fish for $\$2.60$ or
75 g of fish for $\$3.70$

b 750 g of rice for $\$5.60$ or
600 g of rice for $\$4.68$

7 a If 60 g of coriander (A) costs $\$4.20$, what price would 100 g of coriander (B) need to be less than to be the better buy?

b If 222 kg of X-soil costs $\$577.20$, what price would 250 kg of Y-soil need to be more than to become the more expensive purchase?

Dividing a quantity in a given ratio

It is important to be able to divide a quantity, such as an amount of money, into a given ratio.

Examples of dividing a quantity in a given ratio

a Two friends buy a lottery ticket. One of them pays $\$4$ towards the ticket while the other one pays $\$5$ towards the ticket. They agree to split any winnings in the same ratio.
If they win $\$8532$ how much does each of them get?

Total number of parts is: $4 + 5 = 9$ parts.

This questions is asking to divide the quantity $\$8532$ into the ratio $4:5$.

$\therefore$ 9 parts is $\$8532$

$\therefore$ 1 part is $\frac{\$8532}{9} = \$948$

$\therefore$ 4 parts is $\$948 \times 4 = \$3792$

$\therefore$ 5 parts is $\$948 \times 5 = \$4740$. One receives $\$3792$ and the other receives $\$4740$.

b The ratio of girls to boys in a school is $6:7$ and there are 448 boys. How many students are there in the school?

$\therefore$ 7 parts is 448

$\therefore$ 1 part is $\frac{448}{7} = 64$

There are $6 + 7 = 13$ parts in total.

$\therefore$ 13 parts is $13 \times 64 = 832$. Therefore there are 832 students in the school.

c An amount of hay was shared between two farmers who got 420 kg and 700 kg.

(i) What was the total amount of hay shared?

Total $= 420\,\text{kg} + 700\,\text{kg} = 1120\,\text{kg}$

(ii) In what ratio was the hay divided amongst the two farmers?

$$\begin{aligned} 420\,\text{kg} : 700\,\text{kg} &= 420:700 \\ &= 42:70 \\ &= 21:35 \\ &= 3:5 \end{aligned}$$

The ratio is simplified in stages here, but it could have been done in one step by dividing by 140.

We can have more than two terms in a ratio.

d The ratio of sand, gravel and cement for concrete is $1:2:4$. If 24 kg of gravel are used, how much concrete will be produced?

2 parts is 24 kg

1 part is $\frac{24}{2} = 12$ kg

Total number of parts is $1 + 2 + 4 = 7$

7 parts is: $7 \times 12\,\text{kg} = 84\,\text{kg}$

Therefore 84 kg of concrete are produced.

Dividing a quantity in a given ratio

1 Divide the quantities into the ratio given.

a $\$1200$ into the ratio $1:2$.

Total number of parts is ______ $+$ ______ $=$ ______.

$\therefore$ ______ parts is $\$1200$.

$\therefore$ 1 part is $\$$ ______ $\div$ ______ $=$ $\$$ ______.
parts

$\therefore$ 2 parts is $\$$ ______.

$\therefore$ $\$1200$ is divided into the amounts $\$$ ______ and $\$$ ______.

b 700 kg into the ratio $3:11$.

c 360^o into the ratio $5:4$.

2 Harry and Sally purchased a bag containing 36 cookies.
They decide to share them based on how much each contributed to their purchase.
If Harry paid 3 times as much money as Sally did for this purchase, how many cookies does he keep?

Dividing a quantity in a given ratio

3 An amount of money is shared between two people who get $\$450$ and $\$650$.

(i) What was the total amount of the money shared?

(ii) In what ratio was it divided amongst the two people?

4 A barrel of oil was divided into smaller barrels in the ratio $x:5$.
According to this ratio 200 L is divided into $180:20$. Find x.

5 Joan is 10 years older than Fiona. They decide to share $\$400$ in the ratio of their ages.
How much do they each get?
[Hint: if Joan $= x$, then Fiona $= x - 10$.]

Dividing a quantity in a given ratio

6 Reduce these ratios to lowest terms:

a $2:2:4$

b $3:6:12$

c $4:4:4$

d $2:2:4:8$

e $10:5:35:20$

f $6:15:9:27:81$

7 Liam purchases to buy some lotto tickets with 7 friends at work. He pays $\$13$ and everyone else pays $\$1$. The prize is $\$2\,000\,000$.

a Write down what everyone pays towards the lotto tickets as a ratio. [Hint. It starts $1:1:$ ...]

b How much would Liam win if they won the game? How much would everyone else get?

Dividing a quantity in a given ratio

8 Three wheat farmers decide to split 2000 kg of wheat seeds between them in the ratio $\frac{1}{2}:\frac{2}{3}:\frac{5}{6}$.

a Simplify the ratio $\frac{1}{2}:\frac{2}{3}:\frac{5}{6}$. [Hint: find a common denominator.]

b Using the answer to part a find the total number of parts.

c How many kilograms of wheat seed correspond to 1 part?

d How much does each farmer get to the nearest kg?

e About 50 seeds weighing a total of 1.6 g are planted per square metre. What is the size in square metres of the land required to sow all the seeds?

Equivalent ratios with missing terms

Missing terms of equivalent ratios can be found by writing both ratios as fractions.

Examples of finding missing ratio terms

a Solve for x if $x : 5 = 150 : 30$

Write both ratios as fractions first:

$$\frac{x}{5} = \frac{150}{30}$$ If possible, simplify fractions.

$$\frac{x}{5} = \frac{5}{1}$$

method 1

Write both fractions with the same denominator:

$$\frac{x}{5} \times \frac{1}{1} = \frac{5}{1} \times \frac{5}{5}$$

$$\frac{x}{\cancel{5}} = \frac{25}{\cancel{5}}$$ Cancel denominators.

$$\therefore x = 25$$

method 2

Cross multiply fractions:

$$\frac{x}{5} = \frac{25}{5}$$

$$x \times 1 = 5 \times 5$$

$$\therefore x = 25$$

Some questions require you to solve an equation like this:

b Find the pronumeral if $3 : 2 = x : 18$.

Write both ratios as fraction first:

$$\frac{3}{2} = \frac{x}{18}$$

$$3 \times 18 = 2 \times x$$

$$\underset{\div 2}{54} = \underset{\div 2}{2x}$$ Now solve for x by dividing both sides by 2.

$$26 = x$$

$$\therefore x = 26$$

Equivalent ratios with missing terms

Find the value of the missing terms in each of these equivalent ratios.

1 $x : 6 = 2 : 3$

$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

2 $3 : 5 = x : 10$

$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

3 $3x : 4 = 30 : 1$

4 $3 : 2x = 1 : 3$ [Hint: use cross multiplication.]

Equivalent ratios with missing terms

Find the value of the missing terms in each of these equivalent ratios.

5 $15y : 20 \;=\; 1 : 2$

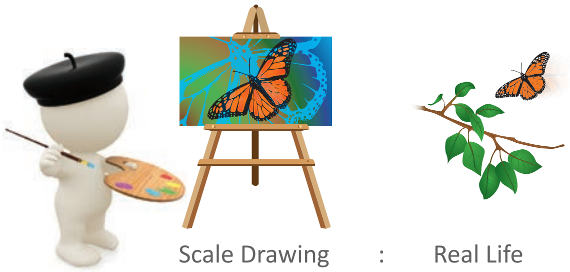
6 $5a : 2 \;=\; 25 : 8$

7 $3x : 5 \;=\; 6 : 6$

8 $y : 6 \;=\; 6 : y$ [Hint: use cross multiplication. Be careful here, there will be 2 answers!]

Scale drawings

A scale drawing is an enlarged or reduced version of a real life object.
All the lengths are reduced or enlarged by the same factor called a scale ratio.
This shows how the drawing's dimensions and the real life dimensions are related.



The left number in the ratio is for the scale drawing and the right number represents Real Life.

Examples with scale drawings

a A scale is such that $2\,\text{cm}$ on the scale drawing represents $5\,\text{m}$ in real life.
Write this scale as a ratio.

As a ratio the scale is $2\,\text{cm} : 5\,\text{m}$.
Remember that $5\,\text{m} = 500\,\text{cm}$.

Before simplifying, then units for both terms must be the same.
The 'cm' was cancelled on each side of the ratio.

$$\begin{aligned} \therefore\ 2\,\text{cm} : 5\,\text{m} &= 2\,\cancel{\text{cm}} : 500\,\cancel{\text{cm}} \\ &= 2 : 500 \\ &= 1 : 250 \end{aligned}$$

Divided both sides by the common factor 2.

So the scale is $1 : 250$.

b In real life an object measures $25\,\text{m}$, and the scale drawing has scale $1 : 2000$.
Find the length of the object on the scale drawing.

$$\begin{aligned} 25\,\text{m} &= 25 \times 100\,\text{cm} \\ &= 2500\,\text{cm} \end{aligned}$$

Divide by 2000 gives $\frac{2500\,\text{cm}}{2000} = 1.25\,\text{cm}$
So on the scale drawing the object has a length of $1.25\,\text{cm}$.

$1\,\text{m} = 100\,\text{cm}$

c Find the scale if $20\,\text{cm}$ on the scale drawing represents $0.5\,\text{mm}$ in real life. The scale as a ratio is:

$$\begin{aligned} 20\,\text{cm} : 0.5\,\text{mm} &= 200\,\cancel{\text{mm}} : 0.5\,\cancel{\text{mm}} \\ &= 200 : 0.5 \\ &= 400 : 1 \end{aligned}$$

$$\begin{aligned} 1\,\text{cm} &= 10\,\text{mm}. \\ \therefore\ 20\,\text{cm} &= 20 \times 10\,\text{mm} \\ &= 200\,\text{mm} \end{aligned}$$

d Find the real life length of an object whose scaled length is $8\,\text{cm}$ on a drawing with scale $2 : 27$.

2 parts is $8\,\text{cm}$.
$\therefore$ 1 part is $4\,\text{cm}$.
$\therefore$ 27 parts is $27 \times 4\,\text{cm} = 98\,\text{cm}$.
The real life length is $98\,\text{cm}$.

Because 2:27 = scale length : real length

Scale drawings

1. Find the scales for each of these:

   a. 1 cm on the scale drawing represents 2 m in real life. [Hint: start 1 cm : 2 m ...]

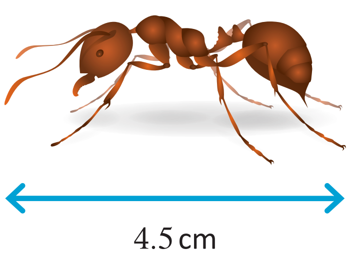
   b. 2 cm on the scale drawing represents 1 m in real life.

   c. 20 cm represents 1 km.

   d. 1 m represents 4 cm.

   e. 1250 mm represents 250 km.

2. Look at the scale drawing of a fire ant. If the scale drawing of the ant has scale $12 : 1$, find the length of the ant in real life.
   [Hint: Let the real life length be x.]



Scale drawings

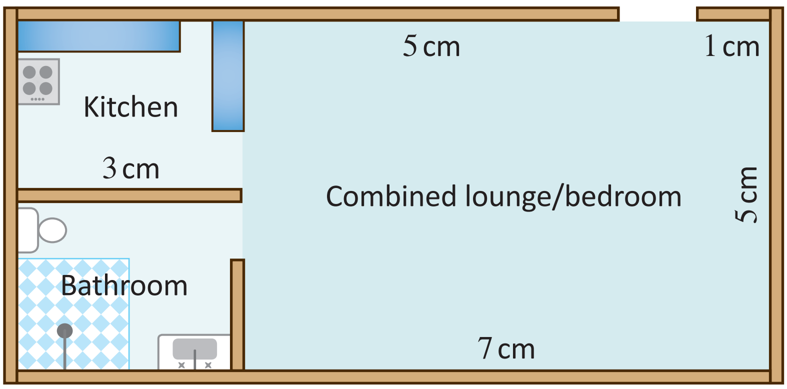
3. Find the real length (x) of some object if the scale drawing has length 6.5 cm and the scale is given by:

   a. $1 : 35$
   b. $13 : 1$
   c. $5 : 16$

4. Find the length on the scale drawing (x) of a real life object of length 4 m if the scale is given by:

   a. $44 : 1$
   b. $1 : 9$
   c. $3 : 40$

5. This diagram is a scale drawing of a studio apartment.



Scale: $1 : 100$

   a. What is the length of the studio apartment on the scale drawing in centimetres?
   b. What is the real life length (x) of the studio apartment in metres?
   c. Find the area of the floor space of the combined lounge/bedroom area.

Maps

Maps are a practical example of scale drawings.

Examples of map scales

a A map scale is $1 : 50\,000$.

(i) How far is the real life distance in kilometres if the map distance is 5 cm?

Since the scale is $1 : 50\,000$ then 5 cm on the map is a real life distance of:

$$\begin{aligned} 50\,000 \times 5\text{ cm} &= 250\,000\text{ cm} \\ &= 2.5\text{ km.} \end{aligned}$$

(ii) If the real life distance is 7 km, how many centimetres is this on the map?

$$\begin{aligned} 7\text{ km} &= 7000\text{ m} \\ &= 700\,000\text{ cm} \end{aligned}$$

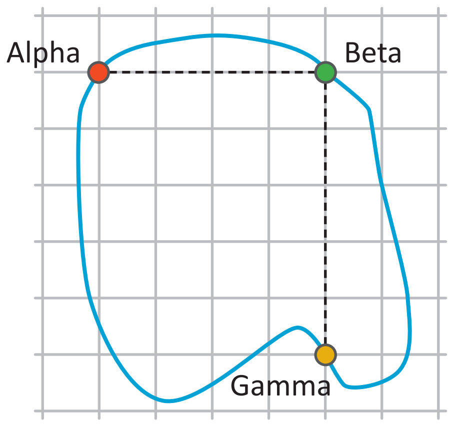
For each $50\,000$ cm in real life it will be 1 cm on the map.

$\therefore$ Therefore $\frac{700\,000\text{ cm}}{50\,000} = 14$ cm

Therefore on the map the distance is 14 cm.

$1\text{ km} = 100\,000\text{ cm}.$

b The real life distance between Alpha Town and Beta Town is 3 km.



(i) Find the scale of this map given that each grid unit is 1 cm.

The scale is the ratio 4 grid units to 3 km.

$$\begin{aligned} 4\text{ cm} : 3\text{ km} &= 4\text{ cm} : 300\,000\text{ cm} \\ &= 4 : 300\,000 \\ &= 1 : 75\,000 \end{aligned}$$

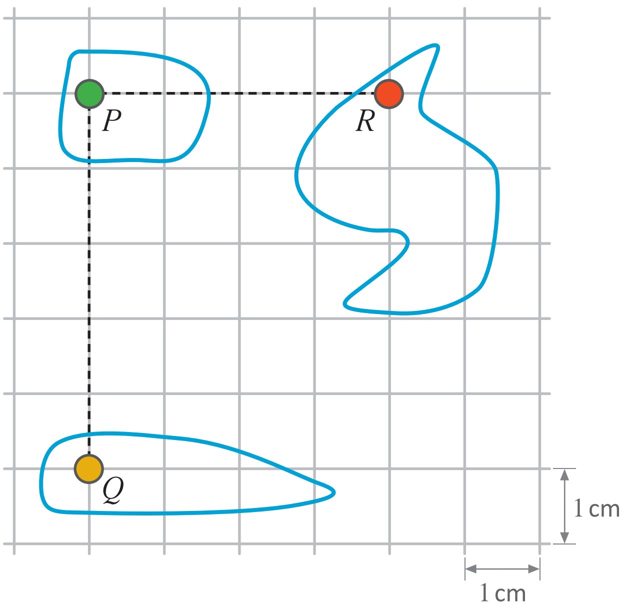
(ii) Using this map scale, find the distance between Beta Town and Gamma Town.
The distance between Beta and Gamma towns on the map is 5 grid units and therefore 5 cm.
Therefore the real life difference is:

$$5 \times 75\,000\text{ cm} = 375\,000\text{ cm}$$

Maps

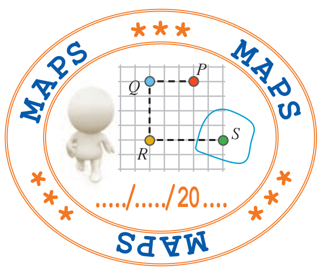
1. A map has a scale of $1:95\,000$. What is the real distance (x) if two points on the map are 12 cm apart?

2. Points P, Q and R are locations on 3 islands for this map. Each grid on the map is 1 cm by 1 cm.



(i) If the real life distance between the two points P and Q is 8 kilometres, find the scale of this map given that each grid unit is 1 cm.

(ii) Using this map scale, find the distance between P and R.



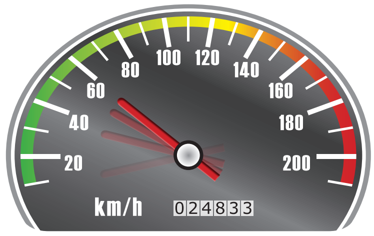
Rates

A rate is a comparison between two quantities of different units.

Imagine that you took 30 steps in 2 minutes, then your rate of walking is 30 steps every 2 minutes.

This is the same as 15 steps every 1 minute, which is 15 steps per minute (written as 15 steps/minute). The forward slash '/' means 'per'.

Travelling 120 kilometres in 2 hours in a car means your rate is 60 km every hour or 60 km per hour (60 km/h).



Examples of rates

a Suppose that a fruit picker picked 42 bins of fruit in 5 days. What is his rate of picking fruit, expressed as bins/day?

42 bins every 5 days	
$\frac{45}{5}$ bins every $\frac{5}{5}$ day	Divide by 5.
$\therefore$ 8.4 bins every 1 day	Simplify.
8.4 bins per day	Using different wording.
8.4 bins/day	Replaced 'per' with the slash symbol '/'.

b A factory makes garments at a rate of 24 garments/h. How many garments are produced in the factory in 40 hours?

24 garments in 1 hour

$\therefore$ 24×40 garments in 40 hours

$\therefore$ 960 garments in 40 hours

c An employee gets a weekly wage of $\$850$ per 5 day week. What is his daily rate of pay?

$\$850$ per 5 days

$\frac{\$850}{5}$ per $\frac{5}{5}$ days

$\$170$ per 1 day

$\$170$ per day

$\$170$/day

We could write 170 $/day but in practise it is usual to put the dollar sign in front of the number.

Rates

1. A boy picks fruit at the rate of 2.5 bins per day. How many bins of fruit does he pick in 12 days?

2. A man works a 35 hour week and his rate of pay is $\$17.50$/h. Find his weekly income.

3. Express the following using basic rates as indicated.

   a. 1920 cars pass a particular traffic light per day. (Cars/hour)

   b. 224 bins of apples were picked by a man over a period of 30 days. (bins/day to the nearest bin)

   c. An employee receives an annual income of $\$82\,420$. If he only works weekdays, find his daily rate of pay. ($\$$/day)

Rates involving flow of liquids

Examples

a A large tank of water holds 5000 L. The water flows out at a rate of 600 mL/s.

(i) How much water flows out of the tank in half an hour to the nearest litre?

600 L/s

$= 600$ L per 1 second

$= 600$ L $\times$ 60 per 1 minute

$= 600$ L $\times$ 60×30 per 30 minutes

Multiply by 30 to get 30 minutes which is half an hour.

$= 1\,080\,000$ mL

$= 1080$ L

(ii) How long does it take to empty this tank to the nearest minute?

Convert 600 mL (to litres) $= \frac{600}{1000}$ L $= 0.6$ L

By dividing by 1000 since 1 L = 1000 mL.

$\frac{5000\text{ L}}{0.6\text{ L}} = 8333.333 \ldots$ s

$\frac{8\,333.333}{60} = 139$ minutes (nearest minute)

b The volume of water in a large water tank on a farm is $325\,000$ L.
If it empties $25\,000$ L over one hour, what is the rate of flow in L/s rounded to one decimal place?

$25\,000$ L per hour $= 25\ 000$ L per 3600 s

$= \frac{25\,000}{3600}$ L/s

$= 6.9$ L/s (1 d.p.)

c Fuel consumption for vehicles is often expressed as a rate of L/100 km. A car travels 900 km and uses 108 L of petrol. Calculate the fuel consumption as a rate with units L/100 km.

108 L per 900 km

$\frac{108}{9}$ L per $\frac{900}{9}$ km

Divided by 9 so it's per 100 km.

$= 12$ L/100 km

d The fuel consumption of a truck is given as 24 L/100 km. If the fuel tank of the truck has a capacity of 648 L, how far can the truck travel on one tank of fuel?

$\frac{648}{24} = 27$

How many times does 24 go into 628?
For each 24 L, the truck travels 100 km.

27×100 km $= 2700$ km

Rates involving flow of liquids

1. A petrol tanker dispenses fuel from a hose at the rate of 50 L/min.

   a. How much petrol is dispensed in 45 minutes?

   b. Calculate the time required to empty a load of $30\,000$ L.

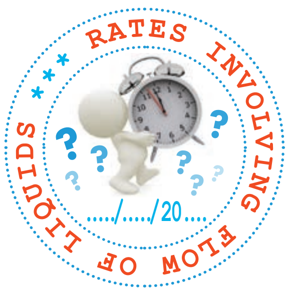
2. The volume of water in a pool is $94\,000$ L. If it empties at a rate of $12\,000$ L in one hour, what is the rate of flow in L/s rounded to one decimal place?

3. A car travels 432 km and uses 35 L of petrol. Calculate the fuel consumption as a rate with units L/100 km. Answer to 1 d.p.

Rates involving flow of liquids

4 The fuel consumption of a truck is 24 L/100 km. If the fuel tank of the truck has a capacity of 648 L, how far (x) can it travel on one tank of fuel?

5 There are two water tanks on a farm. The first one can hold $12\,000$ L and water flows out of the nozzle at a rate of 8.5 L/s. The second one can hold $20\,000$ L and water flows out of the nozzle at a rate of 11.5 L/s. Which tank empties in the shortest time?



Rates involving currency conversions

Example

a. If the currency exchange between US dollars (USD) and British pounds (GBP) is 0.652 GBP/USD,

(i) How much money is $\$2000$ (USD) in British pounds?

$$\$1 \text{ USD} = 0.652 \text{ GBP}$$
$$\therefore \$2000 \text{ USD} = 2000 \times 0.652 \text{ GBP}$$
$$= £1304 \text{ GBP}$$

(ii) How much is $£3000$ (GBP) in United States dollars (USD)?

$$\$1 \text{ USD} = 0.652 \text{ GBP}$$
$$\therefore \frac{1}{0.652} \text{ USD} = 1 \text{ GBP}$$
$$\therefore £3000 = \frac{1}{0.652} \times 3000 \text{ USD}$$
$$= \$4601.23 \text{ (to 2 d.p.)}$$

6. If the currency exchange rate between US dollars (USD) and Australian dollars (AUD) is 0.96 USD/AUD.

a. How much money is $\$5000$ (AUD) in US dollars (USD)?

b. How much is $\$980$ (USD) in Australian dollars (AUD)?

Rates involving currency conversions

7 If the currency exchange rate between British pounds (GBP) and South African rand (ZAR) is 18.51 ZAR/GBP.

a How much money is £$25\,000$ (GBP) in South African rand (ZAR)?

b How much British pounds (GBP) will you need to exchange for R$2\,000\,000$ (ZAR)?

8 Use the exchange rate conversion table below to answer these questions (to 2 d.p.)

Currency	Multiplier exchanging from USD	Multiplier exchanging to USD
Euro (EUR)	~ 0.912	~ 1.096
British pound (GBP)	~ 0.652	~ 1.533
Indian rupee (INR)	~ 63.825	~ 0.0157
Australian dollar (AUD)	~ 1.304	~ 0.767
New Zealand dollar (NZD)	~ 1.398	~ 0.716
Singapore dollar (SGD)	~ 1347	~ 0.742
Japanese yen (JPY)	~ 123.630	~ 0.008

a How much money is ₹$40\,000$ (INR) in US dollars (USD)?

Rates involving currency conversions

8 **b** How much money is $\$260$ (NZD) in US dollars (USD)?

c How much money is $\$200$ (USD) in Japanese yen (JPY)?

d How much money is $\$3600$ (USD) in Singapore dollars (SGD)?

e Use the table to determine which of these amounts in different currencies exchange for the largest amount in US dollars (USD).

$\$349\,000$ (NZD), $\$325\,000$ (AUS), $£163\,500$ (GBP), $\$336\,800$ (SGD), $€229\,000$ (EUR)

Speed

If a car travels at an average speed of 60 km/h for 3 hours, then it follows that in 3 hours the car will have travelled 60 km three times over.

$$60\text{ km/h} \times 3\text{ h} = 180\text{ km}.$$

The general rule is: Distance $=$ Speed $\times$ Time

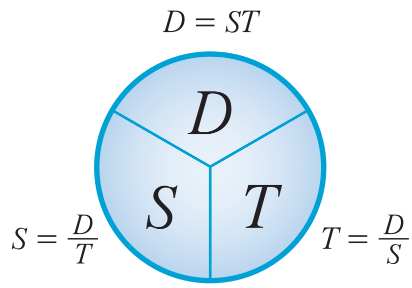
From this formula, the speed or time can also be made the subject of the equation:

$$\frac{\text{Distance}}{\text{Time}} = \frac{\text{Speed} \times \cancel{\text{Time}}}{\cancel{\text{Time}}} \qquad \frac{\text{Distance}}{\text{Speed}} = \frac{\cancel{\text{Speed}} \times \text{Time}}{\cancel{\text{Speed}}}$$

$$\therefore \text{Speed} = \frac{\text{Distance}}{\text{Time}} \qquad \therefore \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

This can be summarized using the formulas:

	Where:
$S = \frac{D}{T}$	$D =$ distance travelled
$D = ST$	$S =$ average speed
$T = \frac{D}{S}$	$T =$ total time taken



Examples using the speed formula

a If a train travels a distance of 1200 km in 15 hours, what is its average speed?

$$S = \frac{D}{T}$$ Substitute the numbers into the equation.

$$= \frac{1200\text{ km}}{15\text{ h}}$$ Keep the units so you know the units at the end.

$$= 80\text{ km/h}$$

b If a car travels at 60 km/h for 3 hours and 90 km/h for 1 hour, find the average speed of the car for the whole trip.

Distance travelled in the first 3 hours: $D = ST = 60(3) = 180$ km

Distance travelled in the last hour: $D = ST = 90(1) = 90$ km

Total distance travelled $= 180$ km $+ 90$ km $= 270$ km

Total time $= 3$ hrs $+ 1$ hrs $= 4$ hrs

Hence the average speed over the whole trip is:

$$S = \frac{D}{T} = \frac{270}{4} = 67.5\text{ km/h}$$

Speed

1. For each of these trips, find the average speed in km/h.

   a. A ferry travels 50 km in two hours.

   b. A small propeller plane flies 400 km in 30 minutes.

   c. A car travels 876 km from Matherton to Ratiato in 9 hours.

2. A train travels at 100 km/h for a period of 3.5 hours. Find the distance travelled by the train.
   [Hint: Use $D = ST$]

3. If a car travels at 80 km/h for 4 hours and 110 km/h for another 4 hours, find the average speed of the car for the 8 hour trip using the following outline:

   a. Find the distance travelled in the first 4 hours.

   b. Find the distance travelled in the last 4 hours.

   c. What's the total distance travelled?

   d. What's the total time period?

   e. Find the average speed of the car for the trip.

Speed

4 A plane flies at a speed of 900 km/h and travels 7000 km non-stop.
How long did the trip take? (nearest minute)

5 A room is 6 m wide. If you run back and forth 15 times in just 3 minutes 15 seconds, what was your average speed in m/s?

6 A woman runs at a speed of 6 m/s for 4 minutes and 30 seconds. What distance did she run?

Speed and different units

It is important to ensure all the units match when using the speed formula. If the speed is in km/h then the time has to be in hours and the distance in km.

Examples

a How far has a bus travelled if it goes at a speed of 54 km/h for a time of 40 minutes?

The basic formula is $D = ST$.

The time must be expressed in terms of hours so the units all match.

$$40 \text{ minutes} = \frac{40}{60} \text{ hours} = \frac{2}{3} \text{ hours}$$

$$\text{Then: } D = 54 \text{ km/h} \times \frac{2}{3} \text{ hours}$$

The units match. These are both in hours.

$$= 54\left(\frac{2}{3}\right) \text{km}$$

$$= 36 \text{ km}$$

b Which is faster, a leopard running at 64 km/h or a bird flying at 15 m/s ?

Convert 64 km/hinto m/s:

$$\frac{64 \text{ km}}{\text{h}} = \frac{64\,000 \text{ m}}{3600 \text{ s}}$$

Remember that $1 \text{ h} = 60 \text{ min} = 60 \times 60 \text{ s} = 3600 \text{ s}$

$$= 17.7... \text{ m/s}$$

Therefore the leopard is running faster than the bird is flying.

c The distance the Earth travels around the Sun in one year is about 940 million kilometres. Assuming one year is 365 days, find the speed of the Earth in km/h.

$$\begin{aligned} 365 \text{ days} &= 365 \times 24 \text{ hours} \\ &= 365 \times 24 \times 3600 \text{ s} \\ &= 31\,536\,000 \text{ s} \end{aligned}$$

$$\begin{aligned} S &= \frac{D}{T} \\ &= \frac{940\,000\,000}{31\,536\,000} \frac{\text{km}}{\text{h}} \\ &= 30 \text{ km/s (to nearest whole km)} \end{aligned}$$

Speed and different units

1. How far has a plane flown if it travels at a speed of 840 km/h for 10 minutes?

2. A shark swims at a speed of approximately 8 km/h. What is this speed in m/s?

3. Which is faster, a paddle boat doing 12 km/h or a cyclist travelling at 4 m/s?
   [Hint: Convert 12 km/h to m/s]

4. In one orbit around the Earth, the moon travels a distance of $2\,410\,000$ km in 27.3 days.
   Show that the speed of the moon around the Earth is 1022 m/s (nearest whole).

Population growth rate

Population Growth Rate (PGR) is the change in population over a specific time period.
It is a percentage of the individuals in the population at the beginning of that period.

The change in population over time is the population at the end minus the population at the beginning of the time period.

$\therefore$ Change in Population = Population at end of time period − Population at beginning of time period

$$\text{PGR} = \frac{\text{Change in population}}{\text{Population at beginning of time period}} \times 100\%$$

Population Growth Rate

Multiply by 100% to make it into a percentage.

Examples of population growth rate calculations

a If the world population in 1990 was $5\,306\,425\,000$ and in 2013 was $7\,095\,217\,980$, find the population growth rate of the world population over this time period.

Change in Population = Population in 2013 − Population in 1990

$$= 7\,095\,217\,980 - 5\,306\,425\,000$$

$$= 1\,788\,792\,980$$

Population Growth Rate,

$$\text{PGR} = \frac{1\,788\,792\,980}{5\,306\,425\,000} \times 100\%$$

$$= 33.7\% \text{ (1 dp)}$$

b If the growth rate remains the same, calculate the world population in 2015.
So the PGR $= 33.7\%$. The population at the beginning of the time period is $7\,095\,217\,980$.

Find 33.7% of this and add in to find the new population.

33.7% of $7\,095\,217\,980 = 0.337 \times 7\,095\,217\,980 = 2\,391\,088\,459$

$\therefore 7\,095\,217\,980 + 2\,391\,088\,459 = 9\,486\,306\,439$

Add the rise in population to the beginning population to get the final population.

Therefore the estimated world population in 2015 is $9\,486\,306\,439$.

Population growth rate

1 If the Mumplegriff's population in 1990 was $17\,169\,000$ and in 2013 was $22\,262\,501$, complete the calculation for the growth rate of Mumplegriff's population in this time period.

Change in Population = Population in 2013 − Population in 1990

$= \boxed{} - \boxed{}$

$= \boxed{}$

Population Growth Rate,

$$\text{PGR} = \frac{\boxed{}}{\boxed{}} \times 100\%$$

$\boxed{}$ % (1 dp)

2 If a country had a population of $120\,000\,000$ in the year 2000 and the population growth rate between 2000 and 2020 was 37%, to find the population in 2020.

Population at beginning of time period = $\boxed{}$.

Find 37% of this and add in to find the new population:

37% of $\boxed{} = 0.37 \times \boxed{}$

$= \boxed{}$

∴ Population in 2020:

∴ $120\,000\,000 + \boxed{} = \boxed{}$

Population growth rate

3 The table shows estimated populations for several countries in 2000 and 2013. Complete the missing parts of the table in the workspace provided below for each one indicated.

Country	Population in 2000	Population in 2013	Change in Population	Population Growth Rate PGR (%)
Australia	19 000 000	22 262 501	326 250	17.2%
Indonesia	206 264 595	251 160 124	c	21.8%
Japan	127 000 000	127 253 075	253 075	0.2%
China	1 242 612 226	b	75 299 292	6.1%
Vietnam	70 000 000	92 477 857	22 477 857	32.1%
United States	281 421 923	316 668 567	35 246 644	12.5%
India	1 040 000 000	1 220 800 359	180 800 359	d
Papua New Guinea	a	6 431 902	544 902	9.3%
Afghanistan	29 863 000	31 108 077	1 245 077	4.2%
North Korea	22 488 000	24 720 407	2 232 407	9.9%
South Korea	47 817 000	48 955 203	113 820	2.4%
Pakistan	140 000 000	193 238 868	53 238 868	38%

a Papua New Guinea – Population in 2000:

b China – Population in 2013:

c Indonesia – Change in Population:

d India – Population Growth Rate:

Data resourced from:
http://en.wikipedia.org/wiki/List_of_countries_by_population_in_2000
https://www.cia.gov/library/publications/the-world-factbook/rankorder/2119rank.html (2013)

Here is a summary of the important things to remember for rates and ratios

Ratios
A ratio compares two quantities and is written using a colon. For example, $2:3$.

Lowest terms and equivalent ratios
A ratio can be reduced to lowest terms by dividing each side of the ratio by common factors.

For example, $3 : 6 = 1 : 2$ by dividing by 3. Two ratios are equivalent if they can be reduced to the same lowest terms.

Reducing decimal ratios
If both numbers in a ratio are finite decimals you can reduce the ratio to lowest terms by first moving the decimal point by the same number of times on the left and right sides of the ratio until you have whole numbers only. Then reduce further by dividing by common factors.

For example, $1.25:0.75 = 125:75 = 5:3$.

Reducing ratios involving fractions
To reduce a ratio involving fractions, first convert them so they have a common denominator. Then cancel the denominator and reduce further.

For example, $\frac{1}{2}:\frac{1}{3} = \frac{3}{6}:\frac{2}{6} = 3:2$.

Best buys and the unitary method
To find the best buy you make one of the numbers in the ratios the same and then compare.
The unitary method is where you make the one of the numbers into a ‘1’.
For example, $2:11 = 1:5\frac{1}{2}$ or $\frac{2}{11}:1$. Then multiply up as required for the problem.

To divide a quantity in a given ratio,

- find the total number of parts,
- then find ‘1’ part,
- then find the answer by multiplying to get the required number of parts.

Scale drawings
A scale drawing is an enlarged of reduced version of a real life object. A scale is a ratio which expresses the enlargement or reduction of the scale drawing. The left number in the ratio is for the scale drawing while the right number represents ‘real life’. Maps are practical examples of scale drawings.

Rates
A rate is a comparison between two quantities such as kilometres per hour, written km/h. The forward slash ‘/’ represents the word ‘per’. Fuel consumptions is often expressed as a rate L/100 km.

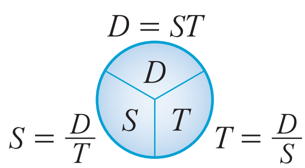
Speed, Distance, Time

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$S = \frac{D}{T}$ $S =$ average speed

$D = ST$ $D =$ distance travelled

$T = \frac{D}{S}$ $T =$ total time taken



Population growth rate
Change in Population = Final Population − Population at beginning of time period

$$\text{Population growth rate (as \%)} = \frac{\text{Change in population}}{\text{Population at beginning of time period}} \times 100\%$$