Angles and Polygons

Curriculum Ready

Mathletics

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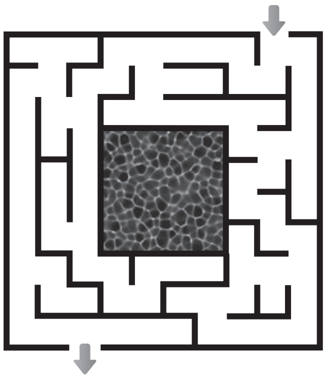
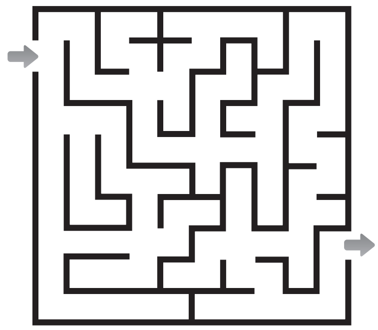
This booklet investigates the internal and external angle properties of polygons.

Investigate these basic mazes to discover an angle property they all have when their entrance and exit points are on opposite sides to each other.

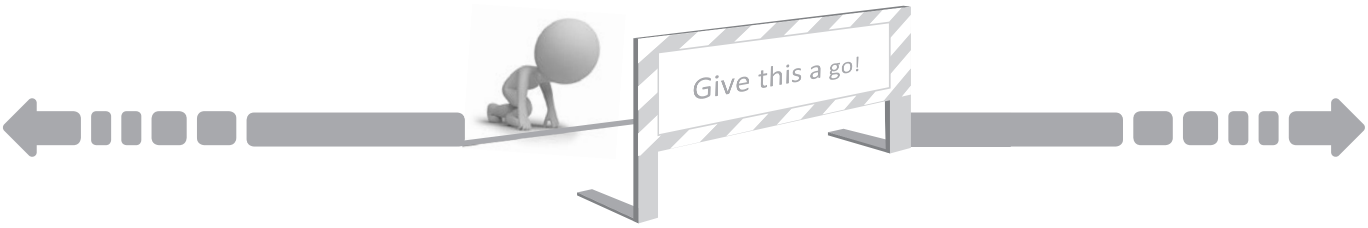
- Let every left-hand turn equal -90°
- Let every right-hand turn equal $+90^\circ$

What is the sum of all the turns made from start to finish for both of these mazes?





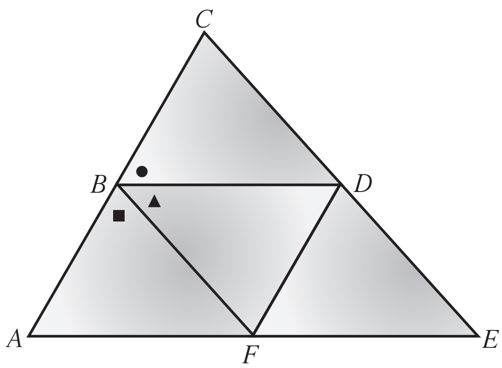
What do you think the sum of all the turns would be if the exit was on the same side as the entrance?



Q Look at the diagram below made using 4 identical scalene triangles.

(i) Mark all the angles that are the same size as the ones shown with a square, triangle and circle.

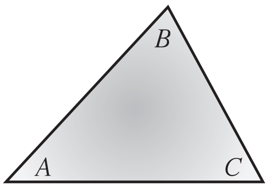
(ii) Name as many different groupings of angles you can that will add together to equal 180° (a straight angle).
For example: $\angle CBD + \angle DBF + \angle ABF = 180^\circ$



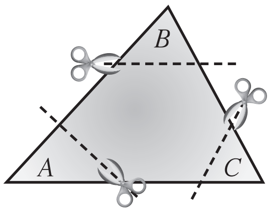
Work through the book for more stuff on angles in polygons

Angle sum of a triangle

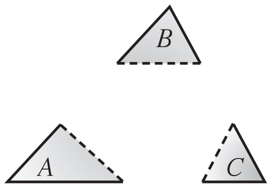
Triangles have this great property where if all the angles are brought together, they form a straight line.



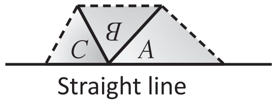
Start with a triangle and label the internal angles A, B and C.



Cut each corner off.



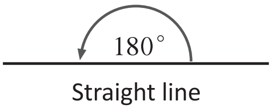
Rearrange the cut-off corners into a straight line.



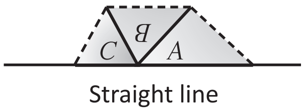
Corner pieces lying perfectly on a straight line.

It works for any triangle. Try it out for yourself!

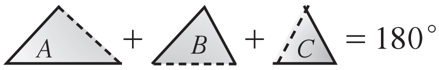
This demonstrates a special property of angles in triangles.



Straight lines form a 180° angle.



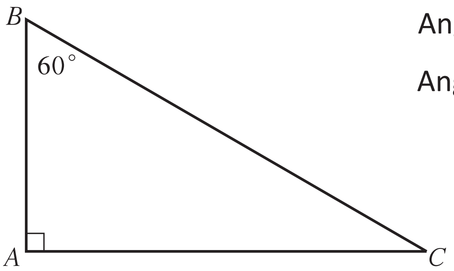
The angles of a triangle form a straight line.



The sum of all the angles in a triangle $= 180^\circ$

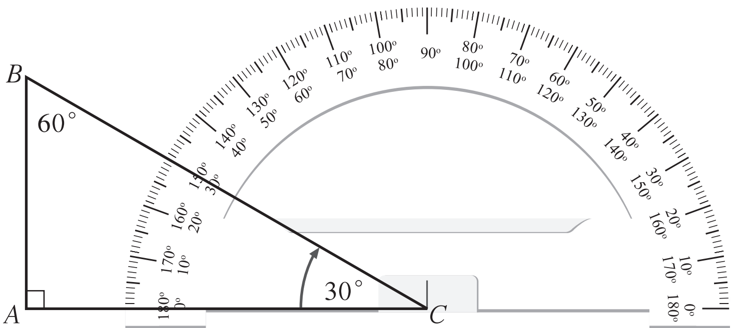
Use the fact that the sum of all the internal angles of a triangle equals 180° to solve these

(i) What is the size of $\angle ACB$ in the triangle below?



$$\begin{aligned}\text{Angle sum of any triangle} &= 180^\circ \\ \text{Angle sum of this triangle} &= \angle ABC + \angle ACB + \angle BAC \\ &= 60^\circ + \angle ACB + 90^\circ \\ &= 150^\circ + \angle ACB \\ \therefore 150^\circ + \angle ACB &= 180^\circ \\ \angle ACB &= 30^\circ\end{aligned}$$

(ii) Measure $\angle ACB$ to show it is 30°.

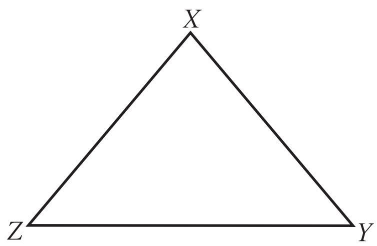


Steps:
1. Put protractor cross-hair at C.
2. Line up side AC and 0°.
3. Measure angle using inside scale.

Angle sum of a triangle

1 Use a protractor to measure each angle in these triangles to the nearest whole degree and show that they add to 180°.

a



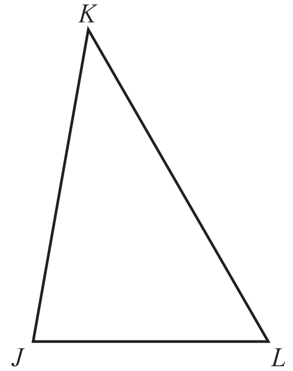
$\angle XYZ =$

$\angle XZY =$

$\angle YXZ =$

Sum $=$

b



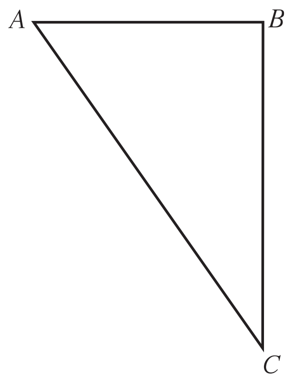
$\angle JKL =$

$\angle JLK =$

$\angle KJL =$

Sum $=$

c



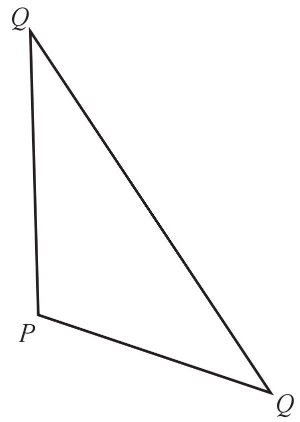
$\angle ABC =$

$\angle ACB =$

$\angle BAC =$

Sum $=$

d



$\angle PQR =$

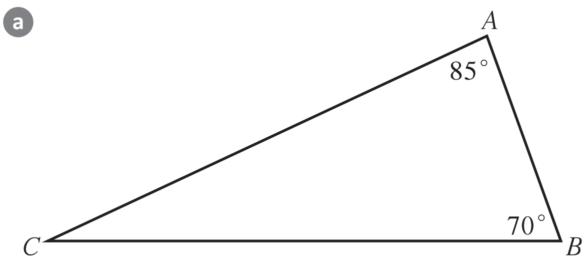
$\angle PRQ =$

$\angle QPR =$

Sum $=$

2 Complete the calculations to find the value of the missing angle in each of these triangles:

a



Angle sum of ΔABC

$= \angle ABC + \angle ACB + \angle BAC.$

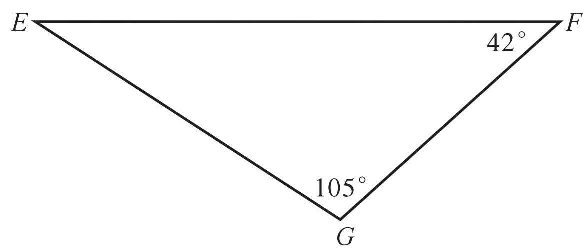
$= 180^\circ$

$\therefore \boxed{\quad}^\circ + \angle BCA + \boxed{\quad}^\circ = 180^\circ$

$\therefore \boxed{\quad}^\circ + \angle BCA = 180^\circ$

$\therefore \angle BCA = \boxed{\quad}^\circ$

b



Angle sum of ΔEFG

$= \angle EFG + \angle EGF + \angle FEG.$

$= 180^\circ$

$\therefore \angle FEG + \boxed{\quad}^\circ + \boxed{\quad}^\circ = 180^\circ$

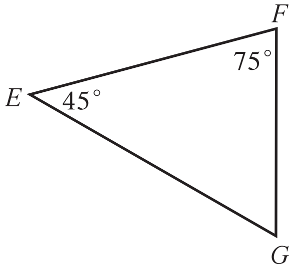
$\therefore \angle FEG + \boxed{\quad}^\circ = 180^\circ$

$\therefore \angle FEG = \boxed{\quad}^\circ$

Angle sum of a triangle

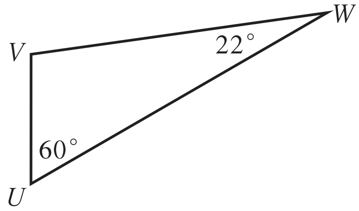
3 Name and calculate the size of the blank angle in these triangles:

a



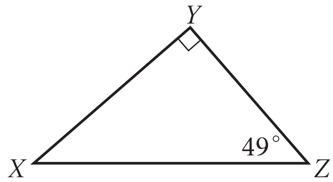
$\angle$ =

b



$\angle$ =

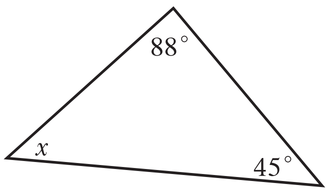
c



$\angle$ =

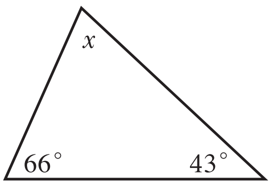
4 The unknown angle in each of these triangles is labelled with the letter x (also called a variable). Calculate the size of each angle represented by the variable x.

a



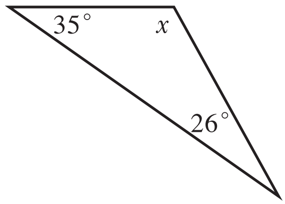
$x =$

b



$x =$

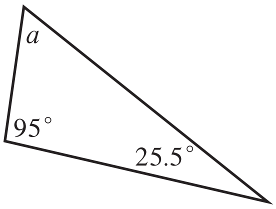
c



$x =$

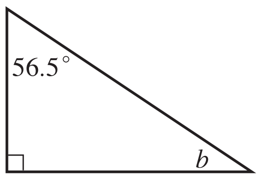
5 Calculate the size of the angle labelled with a letter for these triangles:

a



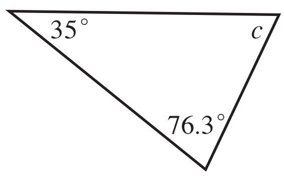
$a =$

b



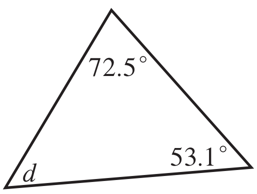
$b =$

c



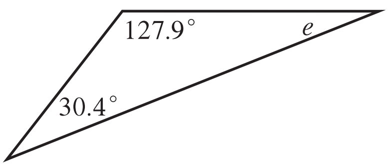
$c =$

d



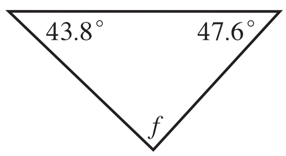
$d =$

e



$e =$

f



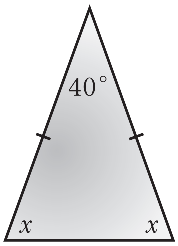
$f =$

Angle sum of a triangle

Some angle sum problems become simple equations to solve.

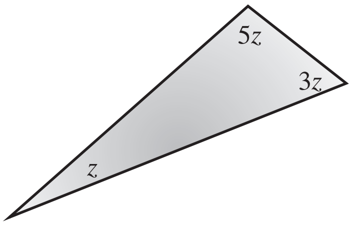
Calculate the value of the variable for each of these triangles

(i) Calculate the value of x in the diagram below:



$40^\circ + x + x = 180^\circ$	Angle sum of a triangle $= 180^\circ$
$40^\circ + 2x = 180^\circ$	Simplify by bringing like terms together
$2x = 140^\circ$	Subtract 40° from 180°
$x = 70^\circ$	Divide 140° by 2 to get the value of x

(ii) Calculate the value of z in the diagram below:

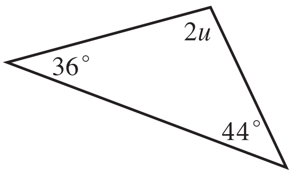


$5z + 3z + z = 180^\circ$	Angle sum of a triangle $= 180^\circ$
$9z = 180^\circ$	Simplify by bringing like terms together
$z = 20^\circ$	Divide 180° by 9 to get the value of z

The size of each angle can be found substituting $z = 20^\circ$ back into each part. $5z = 5 \times 20^\circ = 100^\circ$, $3z = 3 \times 20^\circ = 60^\circ$, $z = 20^\circ$

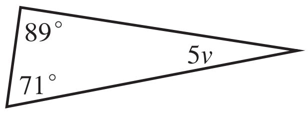
6 Calculate the size of the angle represented by the given variables in these triangles:

a



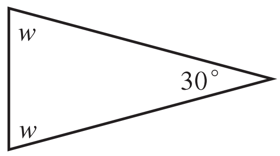
$u =$ ....

b



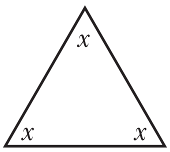
$v =$ ....

c



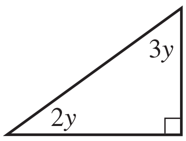
$w =$ ....

d



$x =$ ....

e



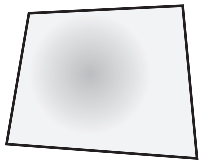
$y =$ ....

f

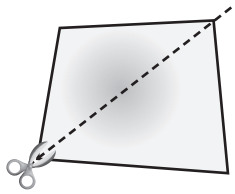
$z =$ ....

Angle sum of a quadrilateral

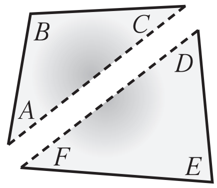
All quadrilaterals can be split into two triangles as a way of showing what their angle sum is.



Start with any quadrilateral



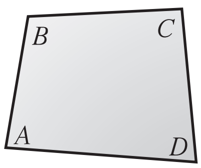
Cut a straight diagonal to make two triangles



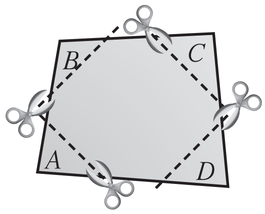
Each triangle has an internal angle sum of 180°

$$\therefore \text{Angle sum of a quadrilateral} = 2 \times \text{angle sum of a triangle}$$
$$= 2 \times 180^\circ$$
$$= 360^\circ$$

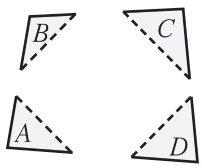
We can also use the angle cut method like we did earlier for a triangle to show they form a revolution.



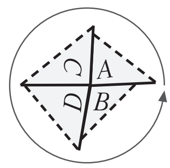
Label the internal angles A, B, C and D.



Cut each corner off.

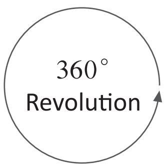


Rearrange the cut-off pieces so all their corners touch.

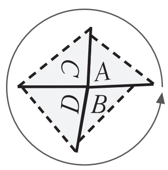


All corners touching form a revolution.

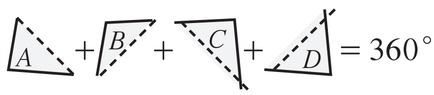
This demonstrates a special property of angles in quadrilaterals.



Angles at a point form a 360° angle.



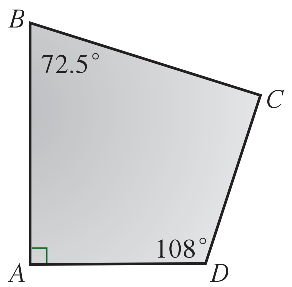
The angles of a quadrilateral form a revolution.



The sum of all the angles in a quadrilateral $= 360^\circ$

Use the fact that the sum of all the internal angles of a quadrilateral equals 360° to solve

What is the size of $\angle BCD$ in the quadrilateral below?

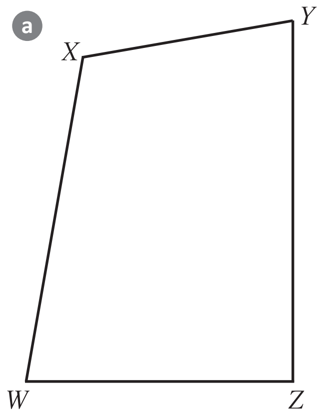


$$\text{Angle sum of any quadrilateral} = 360^\circ$$
$$\text{Angle sum of this quadrilateral} = \angle ABC + \angle BCD + \angle ADC + \angle BAD$$
$$= 72.5^\circ + \angle BCD + 108^\circ + 90^\circ$$
$$= 270.5^\circ + \angle BCD$$
$$\therefore 270.5^\circ + \angle BCD = 360^\circ$$
$$\angle BCD = 89.5^\circ$$

Angle sum of a quadrilateral

1 Use a protractor to measure each angle of these quadrilaterals to the nearest whole degree and show that they add to 360°.

a



$\angle WXY =$ ☐

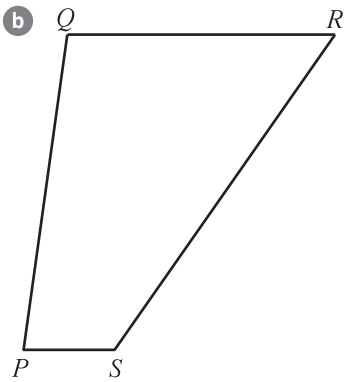
$\angle XYZ =$ ☐

$\angle YZW =$ ☐

$\angle XWZ =$ ☐

Sum = ☐

b



$\angle PQR =$ ☐

$\angle QRS =$ ☐

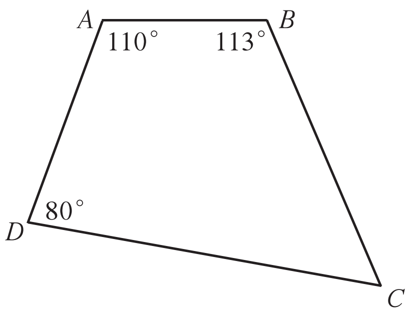
$\angle RSP =$ ☐

$\angle QPS =$ ☐

Sum = ☐

2 Complete these calculations to find the value of the missing angle in each of these quadrilaterals

a



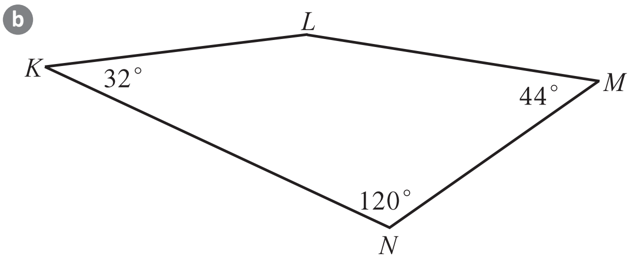
$\angle ABC + \angle BCD + \angle ADC + \angle BAD = 360^\circ$

$\therefore$ ☐° $+ \angle BCD +$ ☐° $+$ ☐° $= 360^\circ$

$\therefore$ ☐° $+ \angle BCD = 360^\circ$

$\therefore \angle BCD =$ ☐°

b



$\angle KLM + \angle LMN + \angle KNM + \angle LKN = 360^\circ$

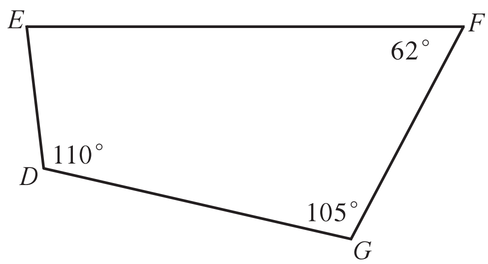
$\therefore \angle KLM +$ ☐° $+$ ☐° $+$ ☐° $= 360^\circ$

$\therefore \angle KLM +$ ☐° $= 360^\circ$

$\therefore \angle KLM =$ ☐°

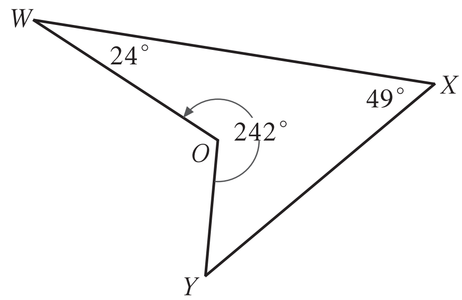
3 Name and calculate the size of the blank angle in these quadrilaterals:

a



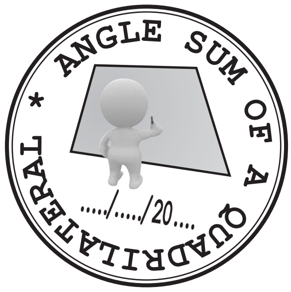
$\angle$ ____ $=$ ____

b



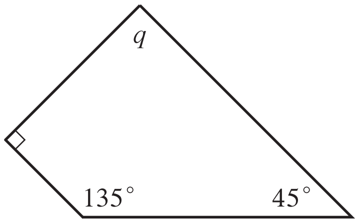
$\angle$ ____ $=$ ____

Angle sum of a quadrilateral



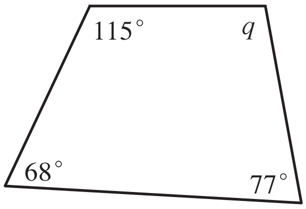
4 The unknown angle in each of these quadrilaterals is labelled with the variable q. Calculate the size of the angle q.

a



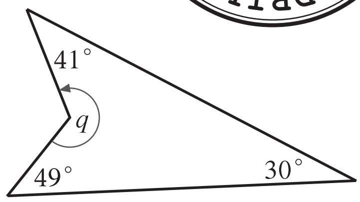
$q =$

b



$q =$

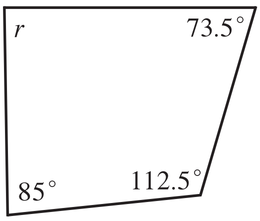
c



$q =$

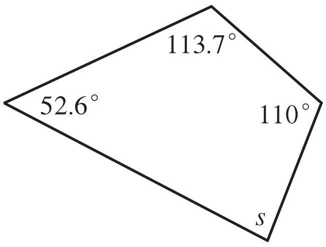
5 Calculate the size of the angle labelled with a variable for these quadrilaterals:

a



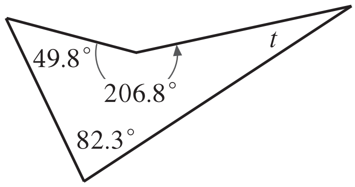
$r =$

b



$s =$

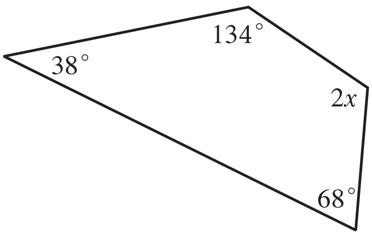
c



$t =$

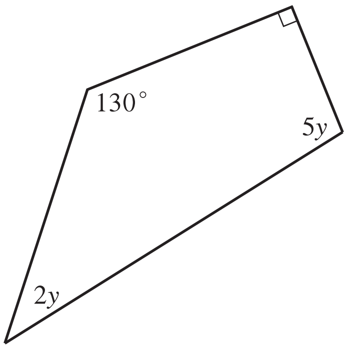
6 Calculate the size of the angle represented by the given variables in these quadrilaterals:
Show all working.

a



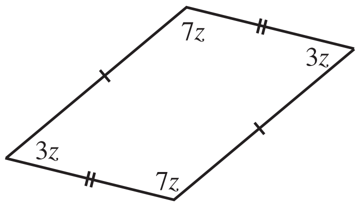
$x =$

b



$y =$

c



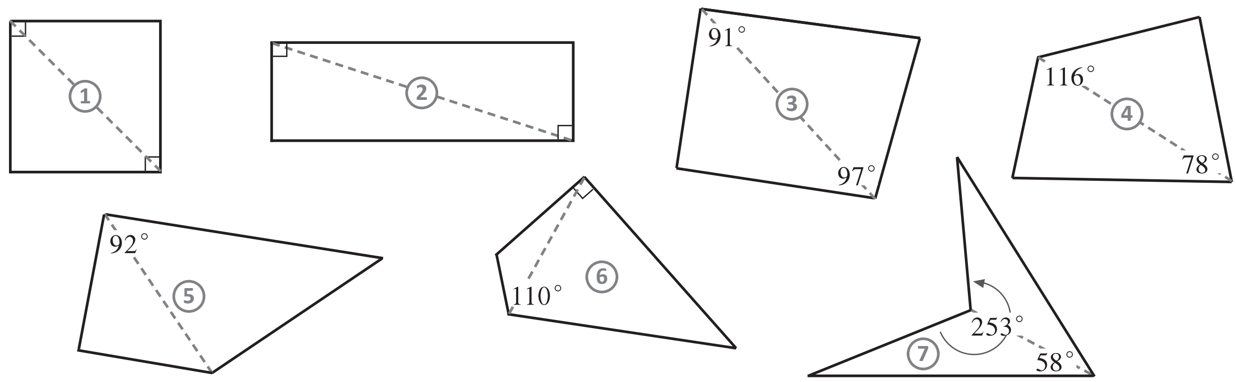
$z =$

Angle sum of a quadrilateral

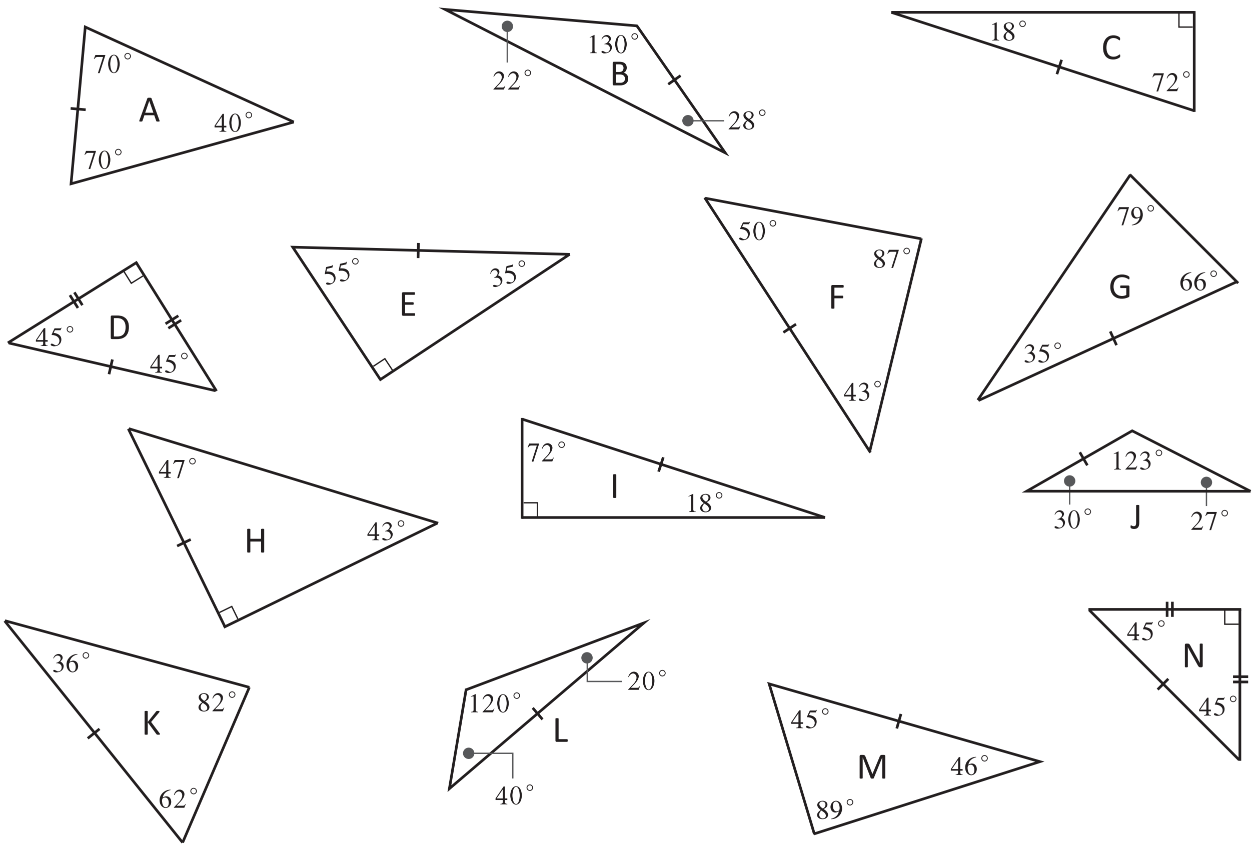
7 Each of these quadrilaterals were divided into two triangles along the dotted line.

Find the matching pair of triangles that were formed after dividing each quadrilateral by comparing the internal angles of the triangles with the angles split by each diagonal.

The sides of the triangles which are a diagonal of a quadrilateral are marked with a single hatch —|—.

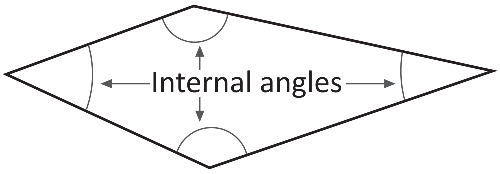


All triangles below are not drawn to scale and some have been rotated.

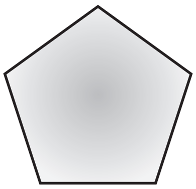


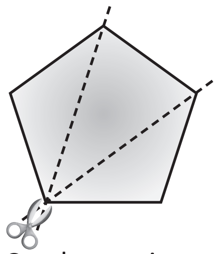
Quadrilateral:	1	2	3	4	5	6	7
Split into triangles:	&	&	&	&	&	&	&

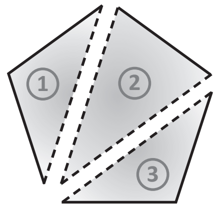
Angle sum of a polygon



All polygons can be split into triangles by drawing all the diagonals possible from one corner (vertex) only.

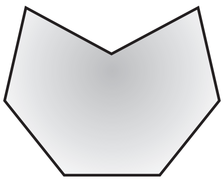

Pentagon
(5 straight sides)

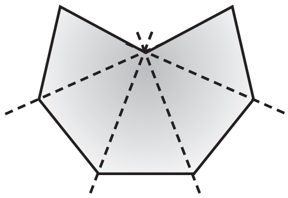

Can be cut into
three triangles

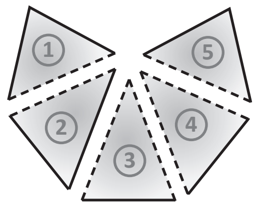


Each triangle has an internal
angle sum of 180°

$$\therefore \text{Internal angle sum of a pentagon} = 3 \times \text{angle sum of a triangle}$$
$$= 3 \times 180^\circ$$
$$= 540^\circ$$


Heptagon
(7 straight sides)


Can be cut into
five triangles



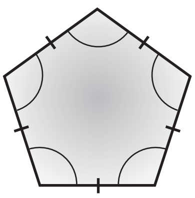
Each triangle has an internal
angle sum of 180°

$$\therefore \text{Internal angle sum of a heptagon} = 5 \times \text{angle sum of a triangle}$$
$$= 5 \times 180^\circ$$
$$= 900^\circ$$

Once the angle sum is known, we can calculate the size of each equal angle in a regular polygon.

Calculate the size of each internal angle for a regular pentagon.

A regular pentagon has 5 equal sides and 5 equal internal angles.



Angle sum of a pentagon $= 540^\circ$ — $3 \times$ angle sum of a triangle

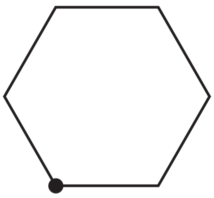
$\therefore$ The size of each angle $= 540^\circ \div 5$ — angle sum divided by the number of internal angles

$= 108^\circ$

Angle sum of a polygon

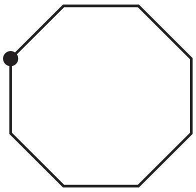
1 (i) Draw all the possible diagonals from the vertex marked with a dot for these polygons.
(ii) Write down the number of triangles each polygon is split into by the diagonals drawn.

a



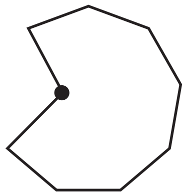
Number of triangles = ☐

b



Number of triangles = ☐

c



Number of triangles = ☐

2 (i) Draw all the possible diagonals from the vertex marked with a dot for these polygons.
(ii) Calculate the internal angle sum of each polygon

a

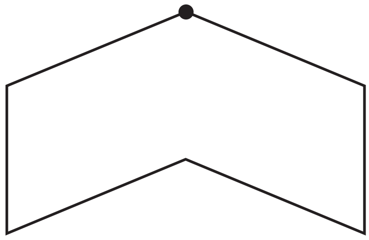
Internal angle sum

= ☐ × angle sum of a triangle

= ☐ × ☐

= ☐

b



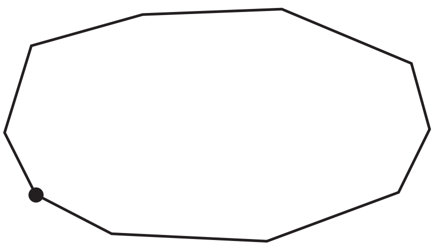
Internal angle sum

= ☐ × angle sum of a triangle

= ☐ × ☐

= ☐

c



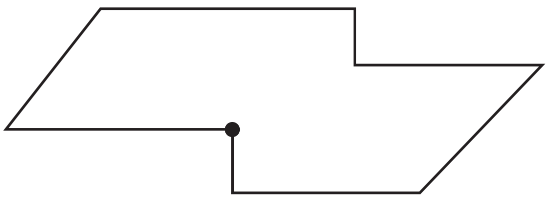
Internal angle sum

= ☐ × angle sum of a triangle

= ☐ × ☐

= ☐

d



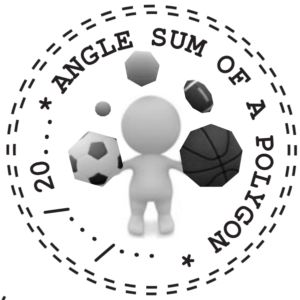
Internal angle sum

= ☐ × angle sum of a triangle

= ☐ × ☐

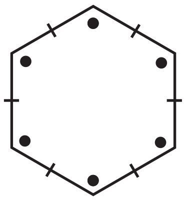
= ☐

Angle sum of a polygon



3 (i) Calculate the angle sum for each of these regular polygons.
(ii) Calculate the size of each internal angle.

a Hexagon

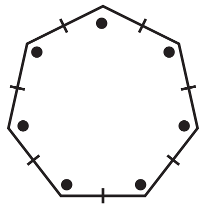


Internal angle sum = ☐

Each internal angle = ☐ ÷ ☐

= ☐

b Heptagon



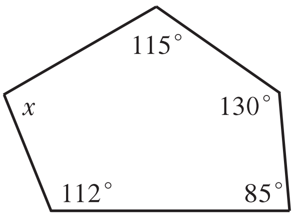
Internal angle sum = ☐

Each internal angle = ☐ ÷ ☐

= ☐ to 1 d.p.

4 Calculate the internal angle sum of these polygons and then calculate the size of the missing angle.

a

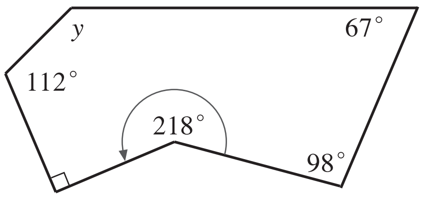


Internal angle sum = $540°$

$\therefore x = 540° - (115° + 130° + 85° + 112°)$

$\therefore x =$ $98°$

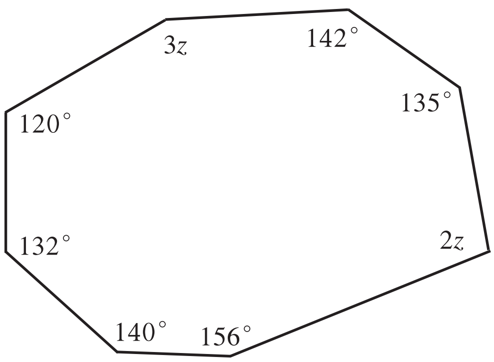
b



Internal angle sum = ☐

$\therefore y =$ ☐

c



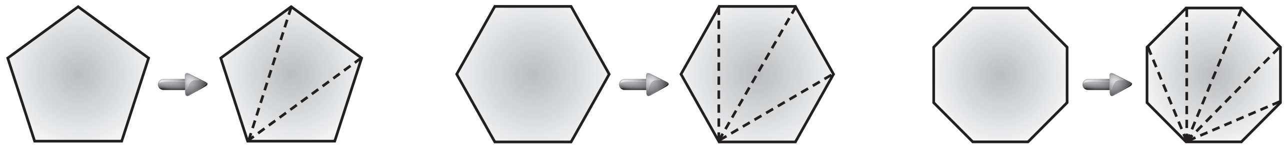
Internal angle sum = ☐

$\therefore z =$ ☐

Internal angle rule for polygons

It can be difficult to draw polygons with a lot of sides and then count the triangles formed.

To make things easier, there is a simple rule you can use from now on for any polygon.



5 sides 3 triangles 6 sides 4 triangles 8 sides 6 triangles

The number of triangles equals two less than the number of sides.

Internal angle sum of a polygon $=$ The number of triangles $\times 180^\circ$

$= ($the number of sides $- 2) \times 180^\circ$

$= (n-2) \times 180^\circ$

$\therefore$ The internal angle sum of any polygon $= (n-2) \times 180^\circ$ (Where $n =$ number of sides)

Use the internal angel rule for these

(i) Calculate the internal angle sum of a heptagon (-sided polygon).

A heptagon has 7 sides, so $n = 7$

$\therefore$ Internal angle sum of a heptagon $= (7-2) \times 180^\circ$ Substitute $n = 7$ into the rule

$= 5 \times 180^\circ$

$= 900^\circ$

To get the size of each internal angle for a regular polygon, just divide by the number of sides (n).

$\therefore$ Each internal angle for a regular polygon $= [(n-2) \times 180^\circ] \div n$

(ii) Calculate the size of each internal angle for a regular hexagon (-sided polygon).

A hexagon has 6 sides, so $n = 6$

$\therefore$ Each internal angle for a regular hexagon $= [(6-2) \times 180^\circ] \div 6$ Substitute $n = 6$ into the rule

$= 720^\circ \div 6$

$= 120^\circ$

Internal angle rule for polygons

1 Complete the internal angle sum calculations for these polygons:

a A hexagon (6-sided polygon)

$n = \boxed{}$

Angle sum $= \left(\boxed{} - \boxed{}\right) \times 180^\circ$

$= \boxed{} \times 180^\circ$

$= \boxed{}$

b A nonagon (9-sided polygon)

$n = \boxed{}$

Angle sum $= \left(\boxed{} - \boxed{}\right) \times 180^\circ$

$= \boxed{} \times 180^\circ$

$= \boxed{}$

2 Calculate the internal angle sum for these polygons:

a A decagon (10-sided polygon)

b A dodecagon (12-sided polygon)

3 Complete the calculations for these regular polygons to find the size of each internal angle:

a A regular octagon (8-sided polygon)

$n = \boxed{}$

Each angle $= \left[\left(\boxed{} - \boxed{}\right) \times 180^\circ\right] \div \boxed{}$

$= \left[\boxed{} \times 180^\circ\right] \div \boxed{}$

$= \boxed{} \div \boxed{}$

$= \boxed{}$

b A regular pentadecagon (15-sided polygon)

$n = \boxed{}$

Each angle $= \left[\left(\boxed{} - \boxed{}\right) \times 180^\circ\right] \div \boxed{}$

$= \left[\boxed{} \times 180^\circ\right] \div \boxed{}$

$= \boxed{} \div \boxed{}$

$= \boxed{}$

4 Calculate the size of each internal angle for these regular polygons (show all working):

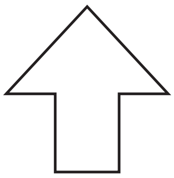
a A regular icosagon (20-sided polygon)

b A regular heptagon (11-sided polygon) to 1 d.p.

Internal angle rule for polygons

5 Calculate the internal angle sum for each of these polygons (show all working):

a



$n =$

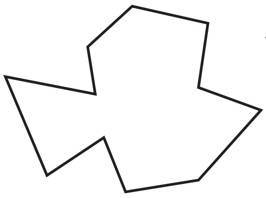
Internal angle sum $=$

b

$n =$

Internal angle sum $=$

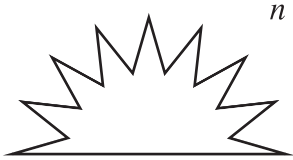
c



$n =$

Internal angle sum $=$

d



$n =$

Internal angle sum $=$

6 We can also calculate how many sides a polygon has from the internal angle sum using this rule:

Number of sides $(n) =$ (Internal angle sum $\div 180^\circ) + 2$

Calculate the number of sides a polygon has with each of these internal angle sums:

a An internal angle sum of 720°

$n = \left(\boxed{} \div 180^\circ \right) + 2$

$= \boxed{}$ sides

b An internal angle sum of 2520°

$n = \left(\boxed{} \div 180^\circ \right) + 2$

$= \boxed{}$ sides

c An internal angle sum of 1800°

d An internal angle sum of 3600°

Internal angle rule for polygons

7 Starting at the top, shade in the one correct path to the bottom to solve this question:

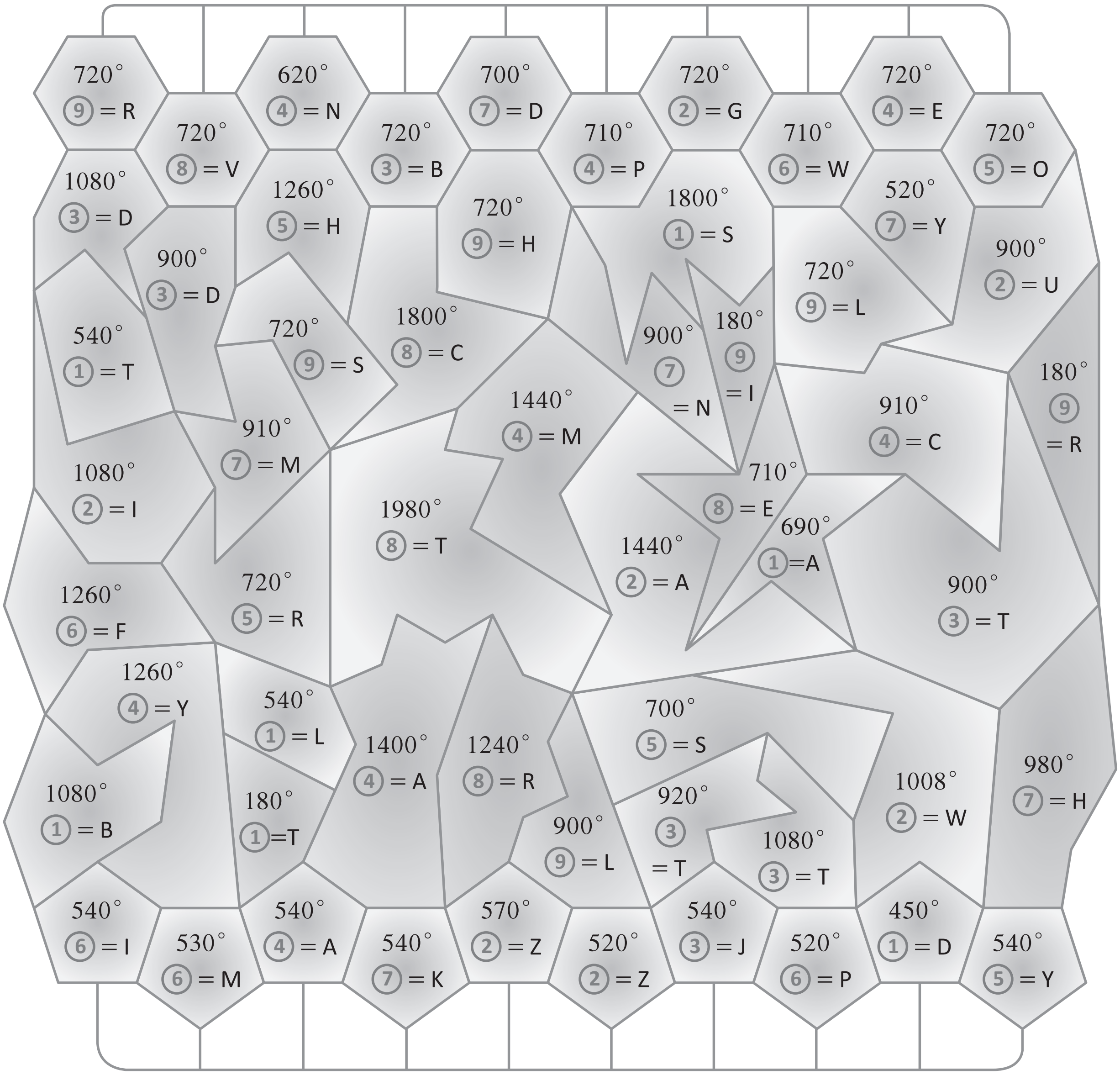
What is the name given to a maze-like structure from Greek mythology that has only one correct path?

Rules to solve the puzzle.

You can:
- only step on polygons containing their correct internal angle sum.
- only move to polygons that share a **side** with the one you are currently on.
- start from any hexagon at the top and finish on any pentagon at the bottom.

Put the letter on each step taken into the matching numbered boxes at the bottom to reveal the answer.

Start anywhere along here

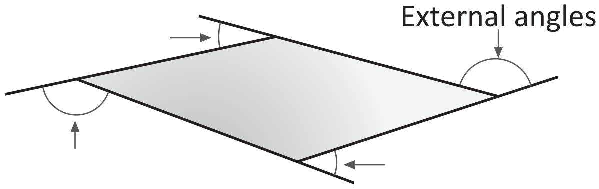


Finish anywhere along here

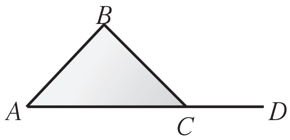
1	2	3	4	5	6	7	8	9

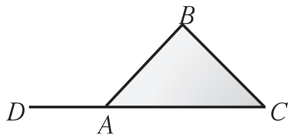
External angle rule for polygons

When you extend a side outside the border of a polygon, you create an external angle.

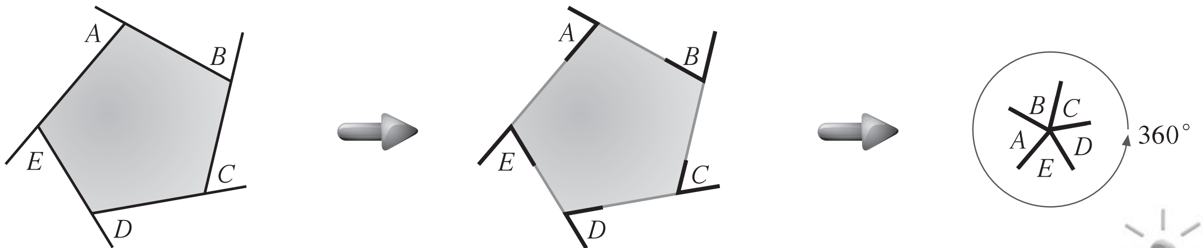


We say extended sides are **produced**. The order of the vertices is important when using this word.

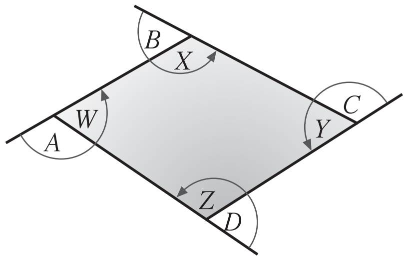
AC is produced to D = 

CA is produced to D = 

All the external angles of any convex polygon add up to 360°.



All the external angles are supplementary (add to 180°) to their adjacent internal angle.



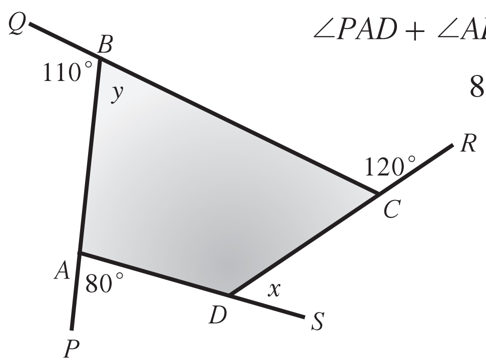
$$A + W = B + X = C + Y = D + Z = 180^\circ$$

Remember: Supplementary angles add to 180°

These properties can be used to find the size of unknown external or internal angles of polygons.

Calculate the value of the variables representing angles in the polygon $ABCDE$ below

BA is produced to P, CB is produced to Q, DC is produced to R and AD is produced to S.



$$\begin{aligned} \angle PAD + \angle ABQ + \angle BCR + \angle CDS &= 360^\circ \\ 80^\circ + 110^\circ + 120^\circ + x &= 360^\circ \\ x &= 360^\circ - 310^\circ \\ x &= 50^\circ \end{aligned}$$

Exterior angle sum = 360°

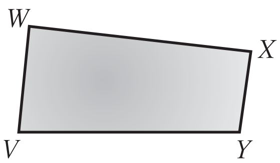
$$\begin{aligned} \angle ABQ + \angle ABC &= 180^\circ \\ 110^\circ + y &= 180^\circ \\ y &= 70^\circ \end{aligned}$$

Supplementary angles

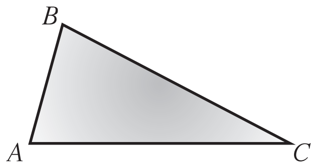
External angle rule for polygons

1 Draw the given side extensions on these polygons:

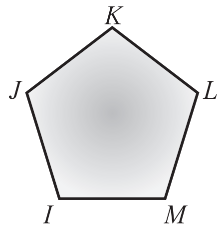
a VY is produced to Z.



b BA is produced to D.

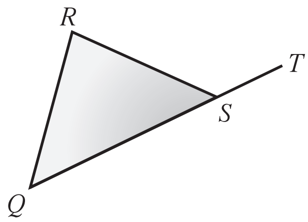


c KJ is produced to N.

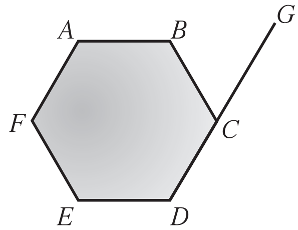


2 Complete these side extension descriptions:

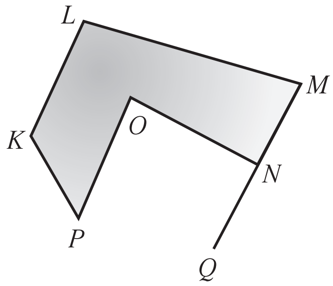
a



b



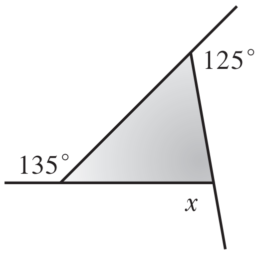
c



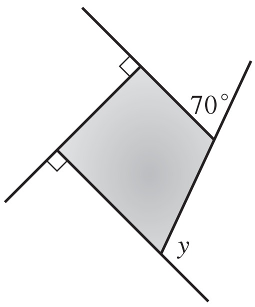
☐ is produced to ☐ ☐ is produced to ☐ ☐ is produced to ☐

3 Calculate the size of the missing external angle in each of these polygons:
Show all working.

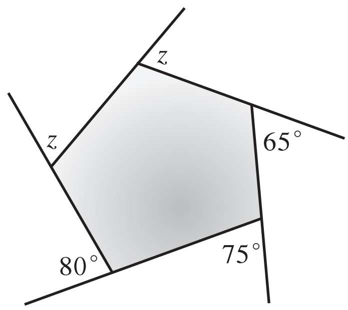
a



b



c



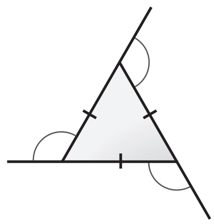
$x =$ ☐ $y =$ ☐ $z =$ ☐

External angle rule for polygons

Each external angle of a regular polygon is the same size.
To calculate the size of each external angle, just divide 360° by the number of angles/sides.

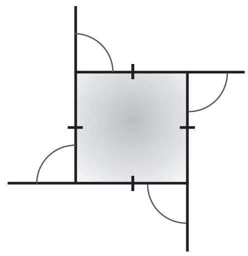
4 Calculate the size of each external angle for these regular polygons accurate to 1 decimal place where required:

a Equilateral triangle



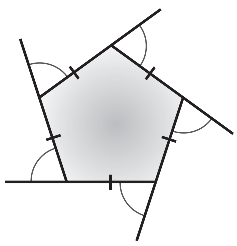
Each external angle = ______

b Square



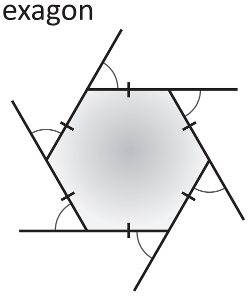
Each external angle = ______

c Regular pentagon



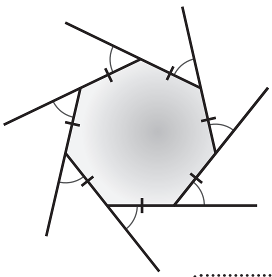
Each external angle = ______

d Regular hexagon



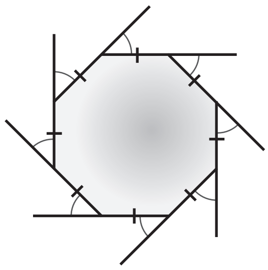
Each external angle = ______

e Regular heptagon



Each external angle = ______

f Regular octagon



Each external angle = ______

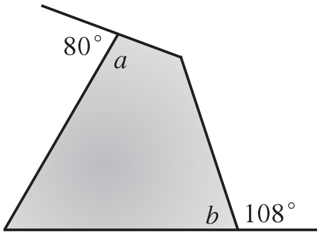
5 Cross out the incorrect term at the end to make these statements true for regular polygons.

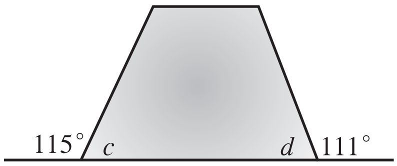
a As the number of sides increases, the size of each external angle increases / decreases.

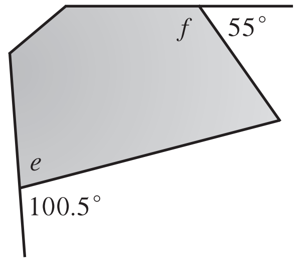
b As the number of sides increases, the size of each internal angle increases / decreases.

External angle rule for polygons

6 Calculate the size of the internal angles marked with variables in each of these polygons:

a


b


c


$a =$ ☐

$b =$ ☐

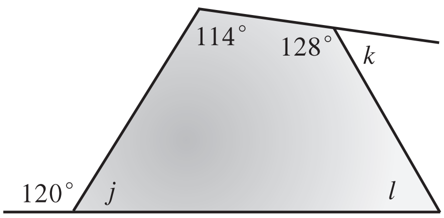
$c =$ ☐

$d =$ ☐

$e =$ ☐

$f =$ ☐

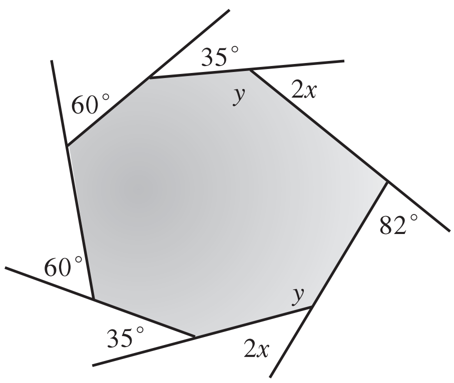
7 Calculate the size of the angles marked with variables in each of these polygons:
(show all working)

a


$j =$ ☐

$k =$ ☐

$l =$ ☐

b


$x =$ ☐

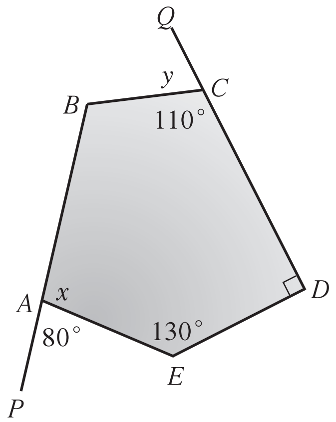
$y =$ ☐

External angle rule for polygons

Earn an awesome passport stamp with these last two challenging questions.

8 (i) Calculate the internal angle sum for this polygon using: angle sum $= (n-2) \times 180^\circ$.

(ii) Calculate the size of the angles marked with variables. (Show all working).



(i)

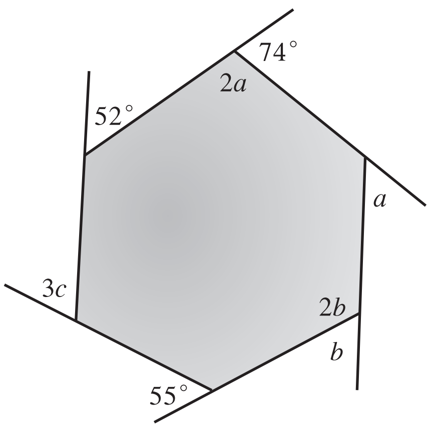
Angle sum =

(ii)

$x =$

$y =$

9 Calculate the size of the variables in the polygon below. Show all your working.



$a =$

$b =$

$c =$

External angle rule for polygons

10 (i) A challenge was given to a class of students to find how many different regular polygons have external angles ranging from 20° through to 30°.
What is the correct answer to this challenge? show all working.

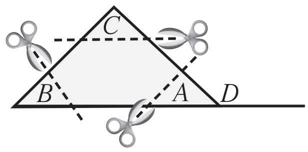
(ii) Write down the number of sides each regular polygon has that fits this criteria along with their external angle size. (round decimal angles to 1 decimal place)

(iii) A further challenge was given to calculate how many sides were needed to form a regular polygon with external angles of exactly 50°.

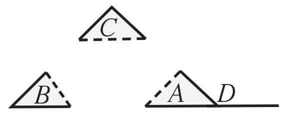
After a few minutes, Tara put her hand up and said that there was no such polygon.
Was Tara correct? Explain your answer including calculations made.

External angle of a triangle

There is a relationship between the internal angles and an external angle for any triangle:

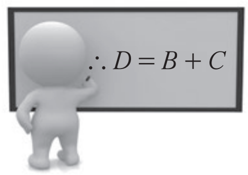


Triangle with one side produced.
Label angles A, B, C and D as shown.

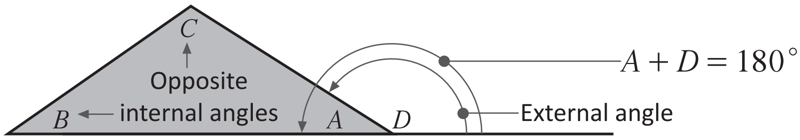


Cut each corner off

Rearrange the cut-off corners B and C to fit into angle D.



Here is a more formal way to show this relationship.
Remember, all external angles are supplementary to their adjacent internal angle.

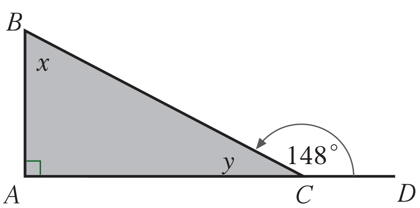


$A + D = 180°$	Adjacent internal and external angles
$\therefore A = 180° - D$	1
$A + B + C = 180°$	Internal angle sum of a triangle $= 180°$
$\therefore A = 180° - (B + C)$	2
$\therefore 180° - D = 180° - (B + C)$	Combining 1 and 2
$\therefore D = B + C$	

The external angle of a triangle = the sum of the two opposite internal angles.

Use the external angle of a triangle rule to find the value of the variables in the diagram

In ΔABC below, the side AC has been produced to D.

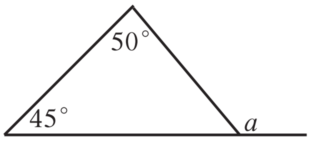


$x + 90° = 148°$	Using exterior angle of a triangle rule
$\therefore x = 148° - 90°$	
$\therefore x = 58°$	
$y + 148° = 180°$	Supplementary angle
$\therefore y = 32°$	

External angle of a triangle

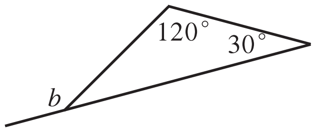
1 Complete the calculations to find the size of the external angles on each of these triangles:

a



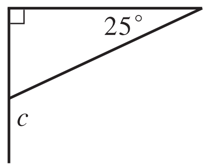
$a =$ ☐ $+$ ☐
$=$ ☐

b



$b =$ ☐ $+$ ☐
$=$ ☐

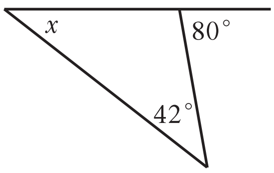
c



$c =$ ☐ $+$ ☐
$=$ ☐

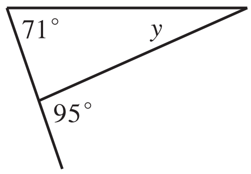
2 Complete the calculations to find the size of the missing internal angles in each of these triangles:

a



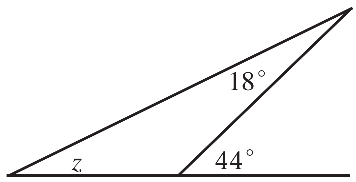
$x =$ ☐ $-$ ☐
$=$ ☐

b



$y =$ ☐ $-$ ☐
$=$ ☐

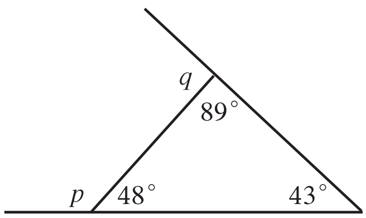
c



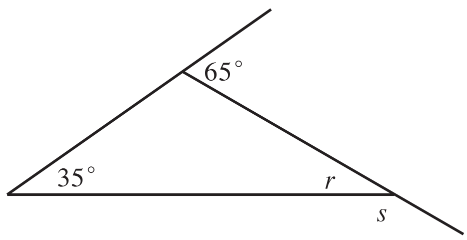
$z =$ ☐ $-$ ☐
$=$ ☐

3 Use the external angle of a triangle rule to calculate the value of the variables shown below:
(show all working)

a



b

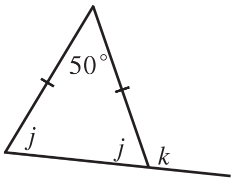


$p =$ ☐ $q =$ ☐ $r =$ ☐ $s =$ ☐

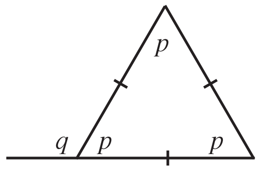
External angle of a triangle

4 Use the external angle of a triangle rule to calculate the value of the variables shown below: (Show all working)

a

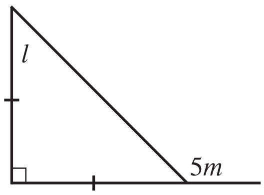


b

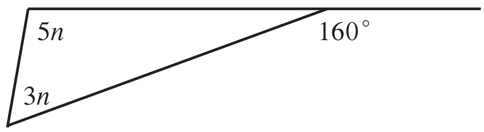


$j =$ ☐ $k =$ ☐ $p =$ ☐ $q =$ ☐

c

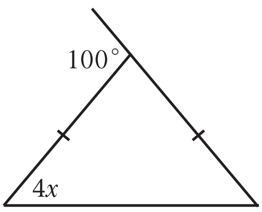


d

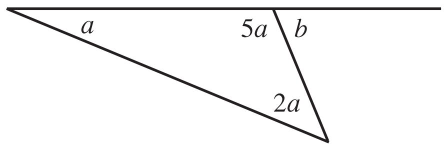


$l =$ ☐ $m =$ ☐ $n =$ ☐

e



f



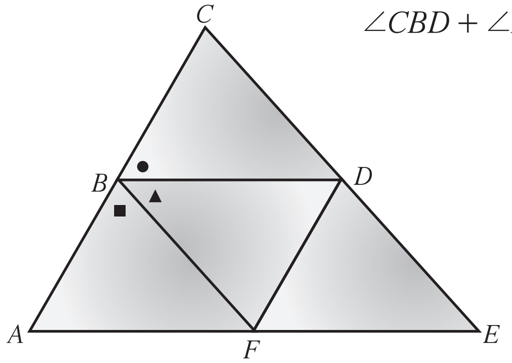
$x =$ ☐ $a =$ ☐ $b =$ ☐

External angle of a triangle

5. Look at the diagram below made using 4 identical scalene triangles.

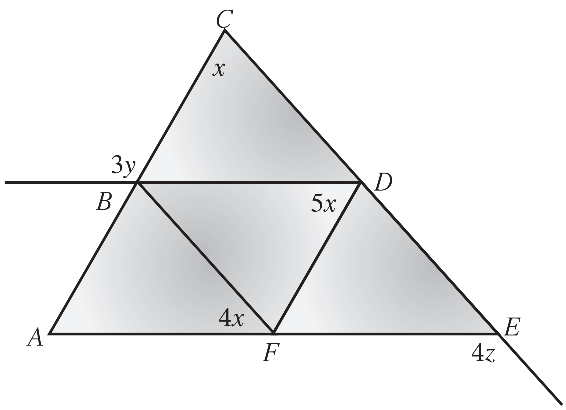
   (i) Mark all the angles that are the same size as the ones shown with a square, triangle and circle.

   (ii) Name as many different groupings of angles as you can that will add together to equal 180° (a straight angle), just like the first three shown:



$\angle CBD + \angle DBF + \angle ABF = 180^\circ$.

   (iii) Earn yourself an awesome passport stamp by finding the value of x, y and z in the diagram below:



$x =$ ______ $y =$ ______ $z =$ ______

Reflection time

Reflecting on the work covered within this booklet:

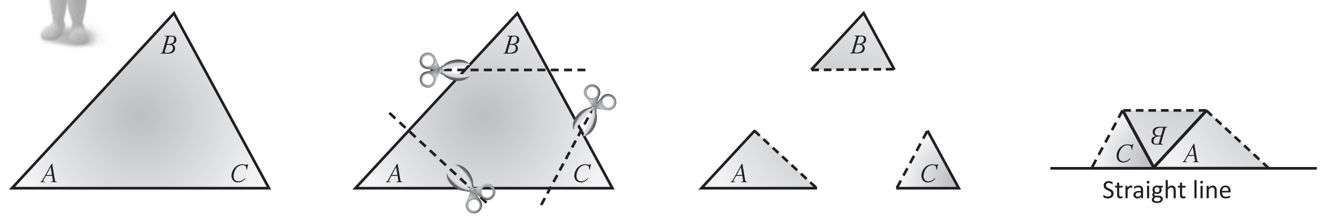
- What useful skills have you gained by learning about the internal and external angles of polygons?

- Write about one or two ways you think you could apply polygon angle calculations to a real life situation.

- If you discovered or learnt about any shortcuts to help with calculating polygon angles or some other cool fact about angles in polygons, jot them down here:

Here is what you need to remember from this topic on angles and polygons

Angle sum of a triangle



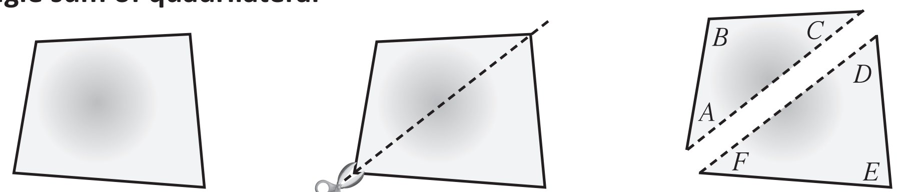
Start with a triangle and label the internal angles A, B and C.

Cut each corner off.

Rearrange the cut-off corners into a straight line.

Corner pieces lying perfectly on a straight line.

Angle sum of quadrilateral



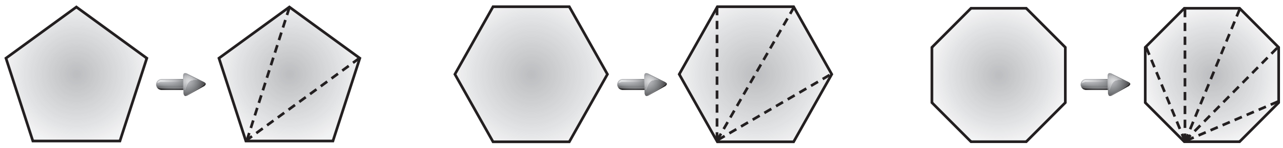
Start with any quadrilateral

Cut a straight diagonal to make two triangles

Each triangle has an internal angle sum of 180°

$\therefore$ Angle sum of a quadrilateral
$= 2 \times$ angle sum of a triangle
$= 2 \times 180^\circ$
$= 360^\circ$

Internal angle sum of a polygon



5 sides → 3 triangles

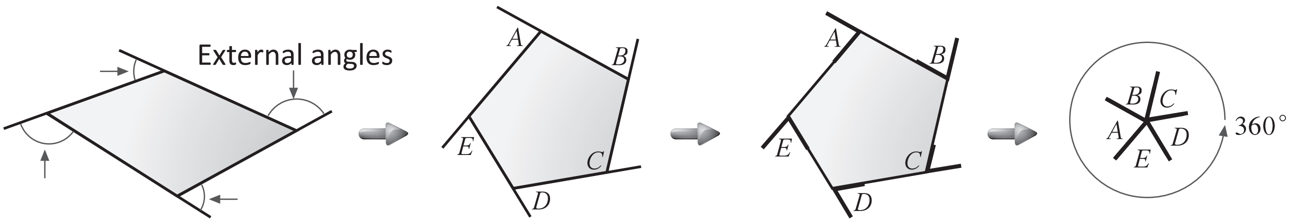
6 sides → 4 triangles

8 sides → 6 triangles

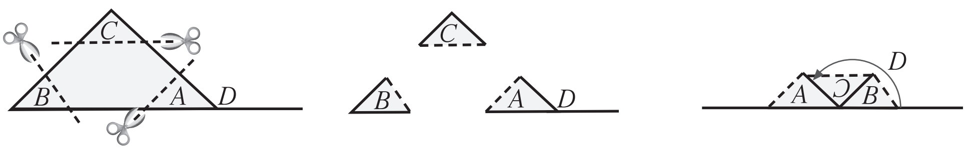
Internal angle sum of a polygon $=$ The number of triangle $\times 180^\circ$
$=$ (the number of sides $- 2) \times 180^\circ$
$= (n-2) \times 180^\circ$

$\therefore$ The internal angle sum of a polygon $= (n-2) \times 180^\circ$

External angle rule for polygons



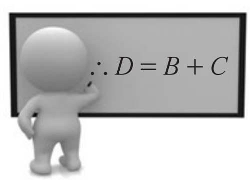
External angle rule for triangle



Triangle with one side produced. Label angles A, B, C and D as shown.

Cut each corner off

Rearrange the cut-off corners B and C to fit into angle D.



Angle sum of a triangle

1. a $\angle XYZ = 50^\circ$
$\angle XZY = 50^\circ$
$\angle YXZ = 80^\circ$
Sum $= 180^\circ$

b $\angle JKL = 40^\circ$
$\angle JLK = 60^\circ$
$\angle KJL = 80^\circ$
Sum $= 180^\circ$

b $\angle ABC = 90^\circ$
$\angle ACB = 35^\circ$
$\angle BAC = 55^\circ$
Sum $= 180^\circ$

c $\angle PQR = 32^\circ$
$\angle PRQ = 38^\circ$
$\angle QPR = 110^\circ$
Sum $= 180^\circ$

2. a $70^\circ + \angle BCA + 85^\circ = 180^\circ$
$145^\circ + \angle BCA = 180^\circ$
$\angle BCA = 35^\circ$

b $\angle FEG + 105^\circ + 42^\circ = 180^\circ$
$\angle FEG + 147^\circ = 180^\circ$
$\angle FEG = 33^\circ$

3. a $\angle EGF = 60^\circ$ b $\angle UVW = 98^\circ$
c $\angle YXZ = 41^\circ$

4. a $x = 47^\circ$ b $x = 71^\circ$ c $x = 119^\circ$

5. a $a = 59.5^\circ$ b $b = 33.5^\circ$ c $c = 68.7^\circ$
d $d = 54.4^\circ$ e $e = 21.7^\circ$ f $f = 88.6^\circ$

6. a $u = 50^\circ$ b $v = 4^\circ$ c $w = 75^\circ$
d $x = 60^\circ$ e $y = 18^\circ$ f $z = 15^\circ$

Angle sum of a quadrilateral

1. a $\angle WXY = 110^\circ$
$\angle XYZ = 80^\circ$
$\angle YZW = 90^\circ$
$\angle XWZ = 80^\circ$
Sum $= 360^\circ$

b $\angle PQR = 98^\circ$
$\angle QRS = 55^\circ$
$\angle RSP = 125^\circ$
$\angle QPS = 82^\circ$
Sum $= 360^\circ$

2. a $113^\circ + \angle BCD + 80^\circ + 110^\circ = 360^\circ$
$303^\circ + \angle BCD = 360^\circ$
$\angle BCD = 57^\circ$

Angle sum of a quadrilateral

2. b $\angle KLM + 44^\circ + 120^\circ + 32^\circ = 360^\circ$
$\angle KLM + 196^\circ = 360^\circ$
$\angle KLM = 164^\circ$

3. a $\angle DEF = 83^\circ$ b $\angle OYX = 45^\circ$

4. a $q = 90^\circ$ b $q = 100^\circ$ c $q = 240^\circ$

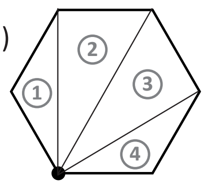
5. a $r = 89^\circ$ b $s = 83.7^\circ$ c $t = 21.1^\circ$

6. a $x = 60^\circ$ b $y = 20^\circ$ c $z = 18^\circ$

7. 1 D & N 2 C & I 3 E & K
4 F & G 5 H & M 6 A & L
7 B & J

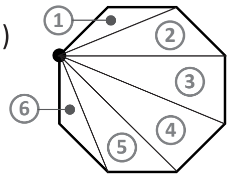
Angle sum of a polygon

1. a (i)



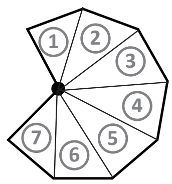
(ii) 4

b (i)



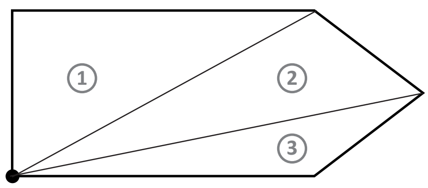
(ii) 6

c (i)



(ii) 7

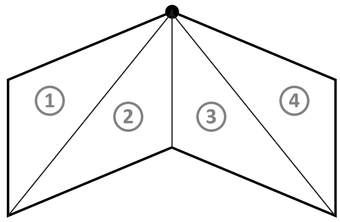
2. a (i)



(ii) Internal angle sum
$= 3 \times$ angle sum of a triangle
$= 3 \times 180^\circ$
$= 540^\circ$

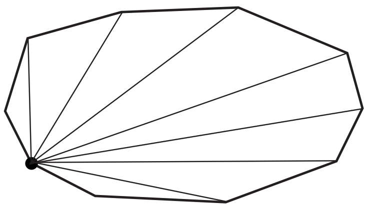
Angle sum of a polygon

2. b (i)



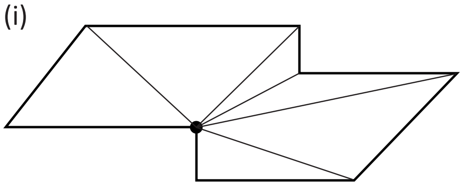
(ii) Internal angle sum
$= 4 \times$ angle sum of a triangle
$= 4 \times 180^\circ$
$= 720^\circ$

c (i)



(ii) Internal angle sum
$= 8 \times$ angle sum of a triangle
$= 8 \times 180^\circ$
$= 1440^\circ$

d (i)



(ii) Internal angle sum
$= 6 \times$ angle sum of a triangle
$= 6 \times 180^\circ$
$= 1080^\circ$

3. a (i) 720° (ii) Each internal angle
$= 720^\circ \div 5$
$= 144^\circ$

b (i) 900° (ii) Each internal angle
$= 900^\circ \div 7$
$= 128.6^\circ$ to 1 d.p.

4. a Internal angle sum $= 540^\circ$ $x = 98^\circ$

b Internal angle sum $= 720^\circ$ $y = 135^\circ$

c Internal angle sum $= 1080^\circ$ $z = 51^\circ$

Internal angle rule for polygons

1. a $n = 6$ Angle sum $= (6 - 2) \times 180^\circ$
$= 4 \times 180^\circ$
$= 720^\circ$

b $n = 9$ Angle sum $= (9 - 2) \times 180^\circ$
$= 7 \times 180^\circ$
$= 1260^\circ$

2. a 1440° b 1800°

3. a $n = 8$ Each angle $= [(8 - 2) \times 180^\circ] \div 8$
$= [6 \times 180^\circ] \div 8$
$= 1080^\circ \div 8$
$= 135^\circ$

b $n = 15$ Each angle $= [(15 - 2) \times 180^\circ] \div 15$
$= [13 \times 180^\circ] \div 15$
$= 2340^\circ \div 150$
$= 156^\circ$

4. a 162° b 147.3°

5. a $n = 7$ Internal angle sum $= 900^\circ$

b $n = 10$ Internal angle sum $= 1440^\circ$

c $n = 11$ Internal angle sum $= 1620^\circ$

d $n = 17$ Internal angle sum $= 2700^\circ$

6. a $n = (720^\circ \div 180^\circ) + 2$
$= 6$ sides

b $n = (2520^\circ \div 180^\circ) + 2$
$= 16$ sides

c 12 sides (dodecagon)

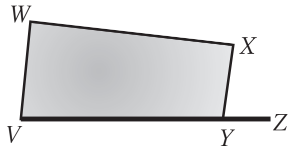
d 22 sides (icosikaidigon)

7.

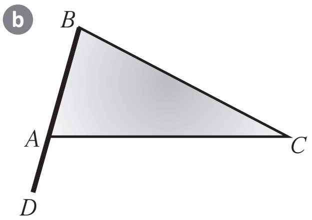
L	A	B	Y	R	I	N	T	H
1	2	3	4	5	6	7	8	9

External angle rule for polygons

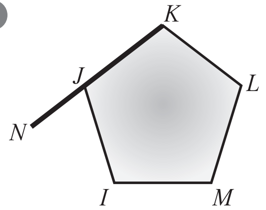
1. a



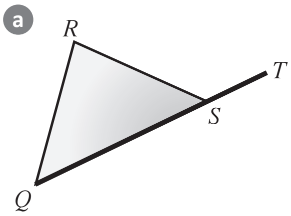
b



c

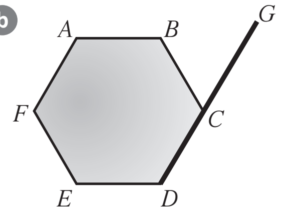


2. a



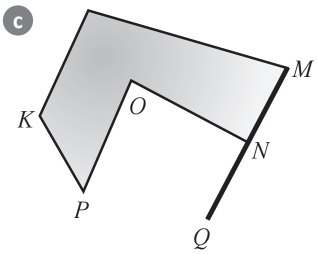
QS is produced to T

b



DC is produced to G

c



MN is produced to Q

3. a $x = 100°$ b $y = 110°$ c $z = 70°$

4. a $120°$ b $90°$ c $72°$
e $51.4°$ f $45°$ d $60°$

5. a decreases b increases

6. a $a = 100°$, $b = 72°$ b $c = 65°$, $d = 69°$ c $e = 79.5°$, $f = 125°$

7. a $j = 60°$, $k = 52°$, $l = 58°$ b $x = 22°$, $y = 136°$

External angle rule for polygons

8. (i) Angle sum $= 540°$ (ii) $x = 100°$, $y = 70°$

9. $a = 53°$ $b = 60°$ $c = 22°$

10. (i) There are seven regular polygons with external angles ranging from $20°$ through to $30°$.

(ii) 12 sides $\longrightarrow$ external angle size $= 30°$
13 sides $\longrightarrow$ external angle size $= 27.7°$ to 1 d.p.
14 sides $\longrightarrow$ external angle size $= 25.7°$ to 1 d.p.
15 sides $\longrightarrow$ external angle size $= 24°$
16 sides $\longrightarrow$ external angle size $= 22.5°$
17 sides $\longrightarrow$ external angle size $= 21.2°$
18 sides $\longrightarrow$ external angle size $= 20°$

(iii) correct

External angle of a triangle

1. a $a = 45° + 50° = 95°$ b $b = 120° + 30° = 150°$
c $c = 90° + 25° = 115°$

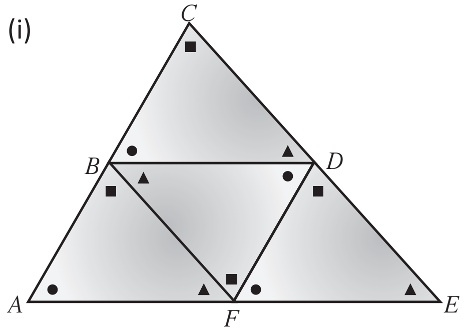
2. a $x = 80° - 42° = 38°$ b $y = 95° - 71° = 24°$
c $z = 44° - 18° = 26°$

3. a $p = 132°$, $q = 91°$ b $r = 30°$, $s = 150°$

4. a $j = 65°$, $k = 115°$ b $p = 60°$, $q = 120°$
c $l = 45°$, $m = 27°$ d $n = 20°$
e $x = 12.5°$ f $a = 22.5°$, $b = 67.5°$

External angle of a triangle

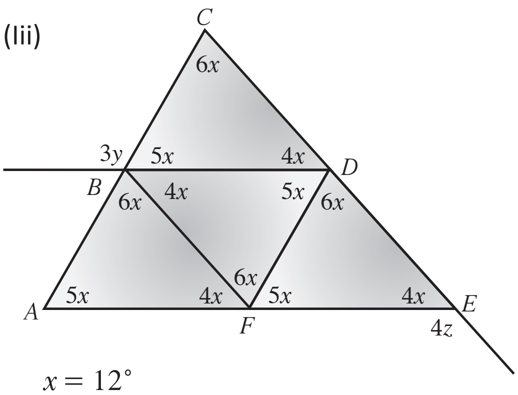
5. (i)



(ii) There are MANY combinations. Here are a four:

$\angle ABF + \angle BFA + \angle FAB = 180^\circ$
$\angle ACE + \angle CEA + \angle CAE = 180^\circ$
$\angle ABF + \angle FBD + \angle BDF = 180^\circ$
$\angle BDC + \angle DFE + \angle DEF = 180^\circ$

(Iii)



$x = 12^\circ$
$y = 40^\circ$
$z = 33^\circ$

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