Angles

Curriculum Ready

Mathletics

www.mathletics.com

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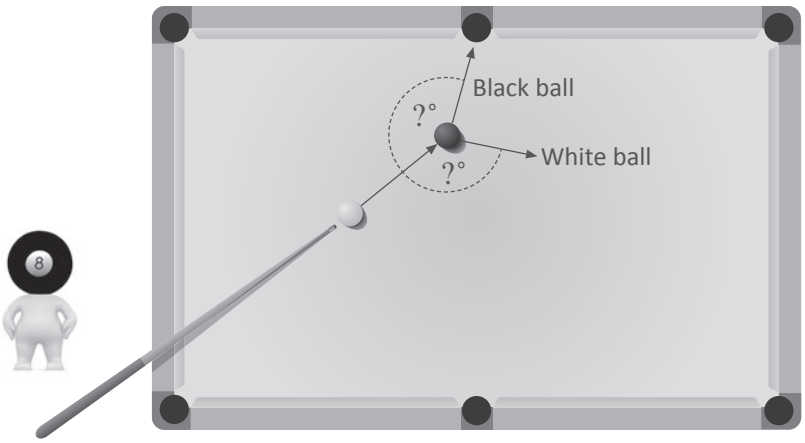
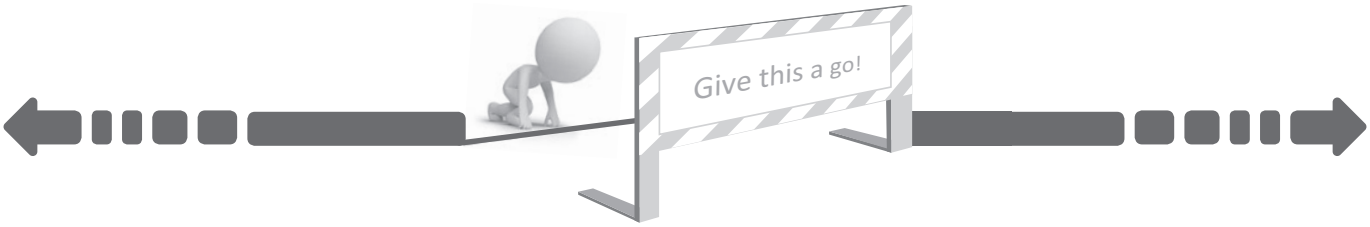
Angles measure the amount of turn in degrees between two lines that meet at a point.

Many games are based on interpreting and using angles such as pool, snooker and billiards.

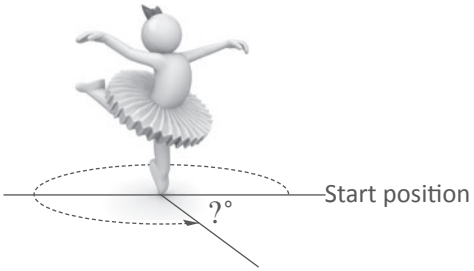


Write down some other sports/games that you can think of that require an understanding of angles:



Q While performing a circular ballet move, Janet turned the first half easily and then with some extra effort, made it $\frac{5}{6}$ of the remaining way around. How many degrees was Janet away from completing the full circle?

Hint: half a circle is 180 degrees.



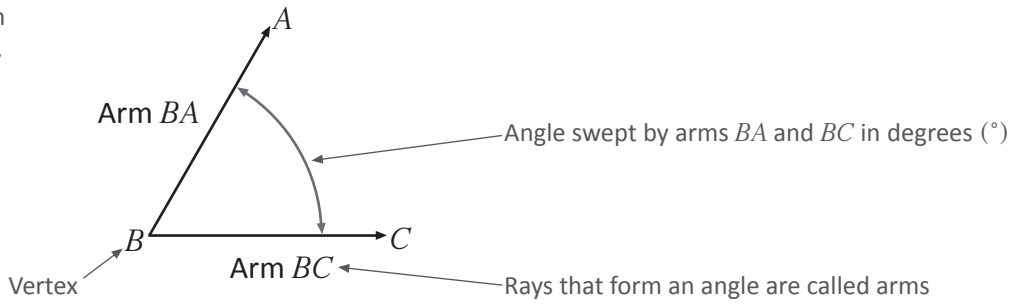
Work through the book for a great way to solve this

Parts of an angle

Angles are formed when two straight rays extend from a common point.

The amount of rotation 'swept' from one arm to the next in degrees is how they are measured

Rays are straight lines with an arrow on one end only.

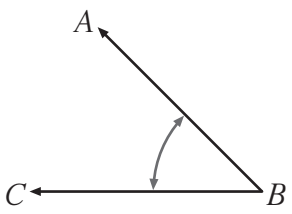


Naming angles

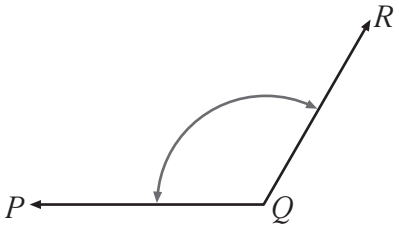
These two methods of naming use the symbol ∠ in front to mean 'angle'.

Name these two angles:

(i)



(ii)



Method 1 $\angle ABC$ or $\angle CBA$ $\angle PQR$ or $\angle RQP$

The letter at the vertex is **always** written in the middle

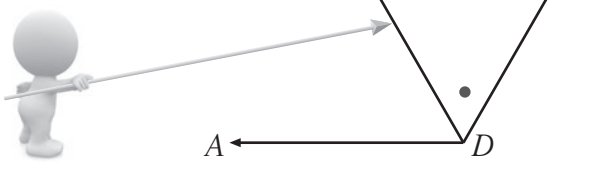
Method 2 $\angle B$ $\angle Q$

For angles like these, you can just use the letter at the vertex

If there is more than one angle at the same point, you must use method 1 to reference the angle properly.

Name these angles marked with a dot:

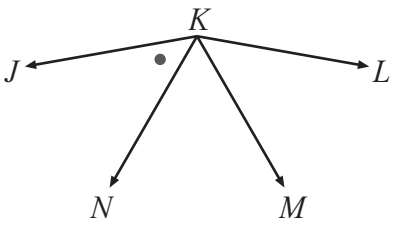
(i)



BD is shared by both angles (Common arm)

$\angle BDC$ or $\angle CDB$

(ii)

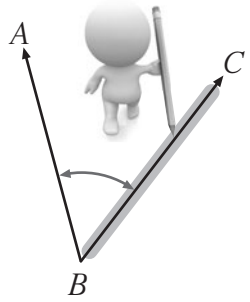


$\angle JKN$ or $\angle NKJ$

Parts of an angle

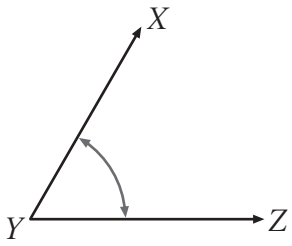
1 Highlight the section of the angle that matches the label underneath.

a



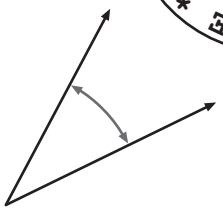
Arm BC

b



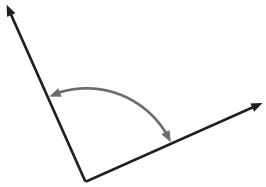
Arm YX

c



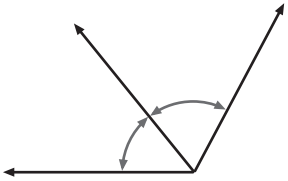
The vertex

d



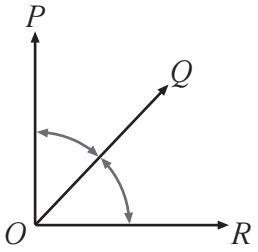
The angle swept

e



The arm shared by both angles

f

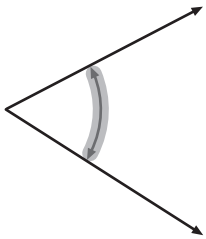


The angle swept by the arms OP and OQ

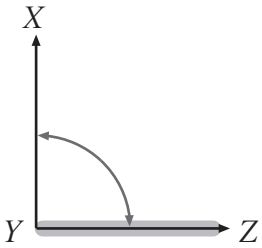
2 Write down the parts of the angles that have been highlighted below.

a

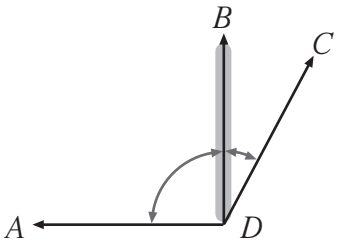
b



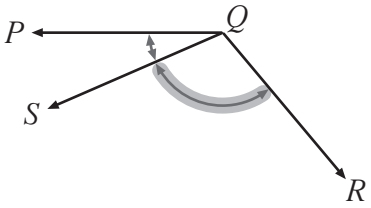
c



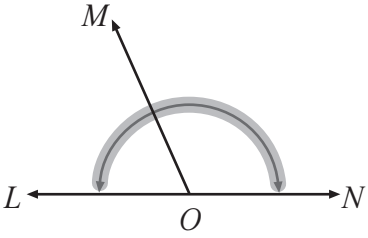
d



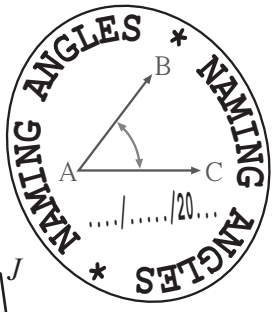
e



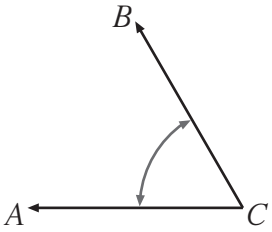
f

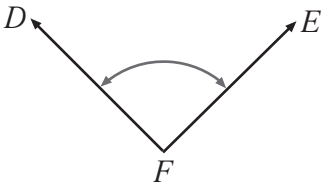


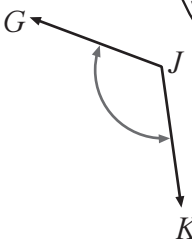
Naming Angles



1 Name each of these angles.

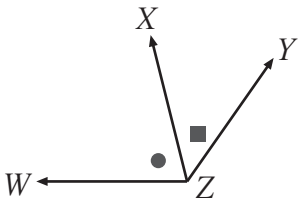
a 

b 

c 

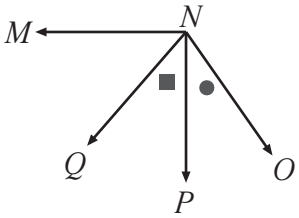
2 Name each of the angles marked with: (i) A dot ●

(ii) A square ■

a 

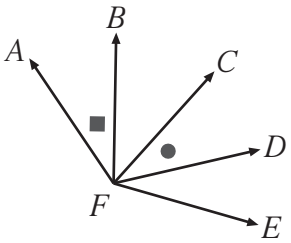
(i) ●

(ii) ■

b 

(i) ●

(ii) ■

c 

(i) ●

(ii) ■

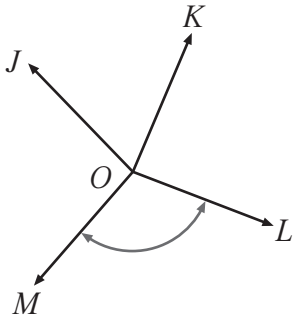
3 Name the arm common to both marked angles in question 2, (write 'no arm common' if there isn't one).

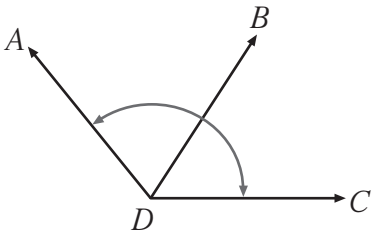
a

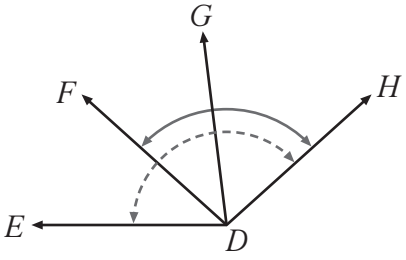
b

c

4 Name the angles indicated in each of these:

a 

b 

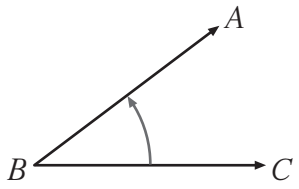
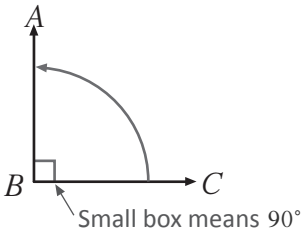
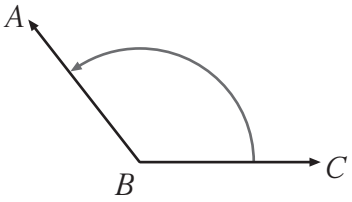
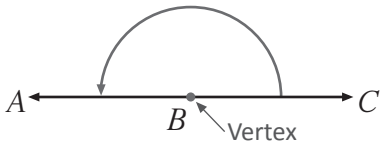
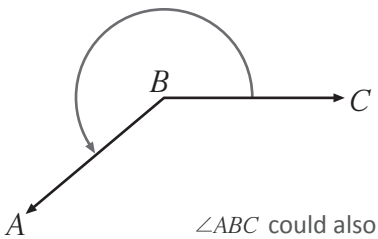
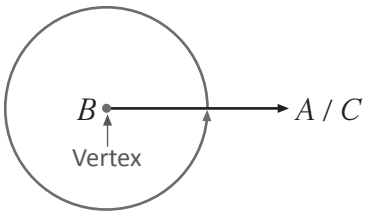
c 

=

=

Angle types

This table shows how angles are classified by their size.

Picture	Size	Name
	Between $0°$ and $90°$ or $0° < \angle ABC < 90°$	**Acute angle**
	Exactly $90°$ or $\angle ABC = 90°$	**Right angle**
	Between $90°$ and $180°$ or $90° < \angle ABC < 180°$	**Obtuse angle**
	Exactly $180°$ or $\angle ABC = 180°$	**Straight angle**
	Between $180°$ and $360°$ or $180° <$ reflex $\angle ABC < 360°$ $\angle ABC$ could also be an obtuse (or acute) angle, so include the word reflex in front	**Reflex angle**
	Exactly $360°$ or $\angle ABC = 360°$	**Reflex angle** or **Full rotation**

Angle types

1. Sketch and label angles that match each of these descriptions:

   a. Obtuse angle $\angle XYZ$

   b. Acute $\angle PQR$

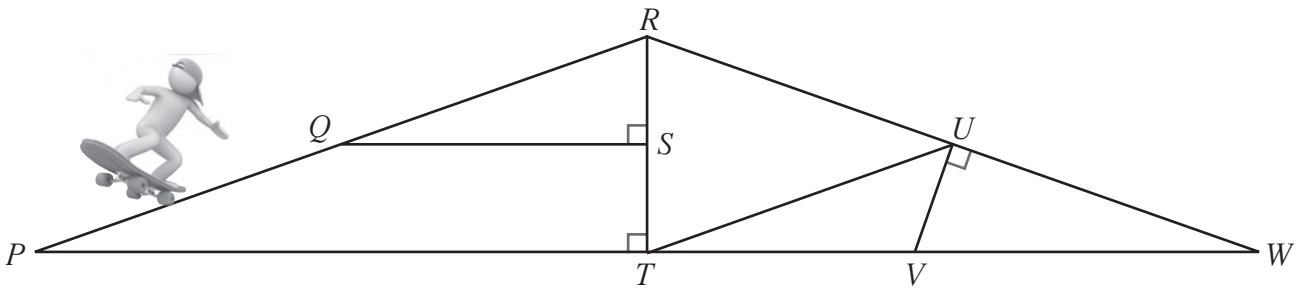
   c. Right-angle $\angle MLN$
   (Hint: remember the box)

   d. Reflex $\angle GUP$

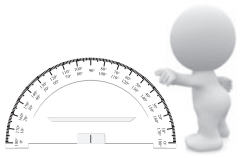
   e. Full revolution $\angle JKL$

   f. Straight $\angle DEF$

2. Fill in the table below with four angles you can find matching the types in the diagram below:



Acute angle	Right angle	Obtuse angle	Straight angle	Reflex angle
$\angle RQS$	$\angle RSQ$	$\angle PRW$	$\angle PQR$	reflex $\angle PRW$



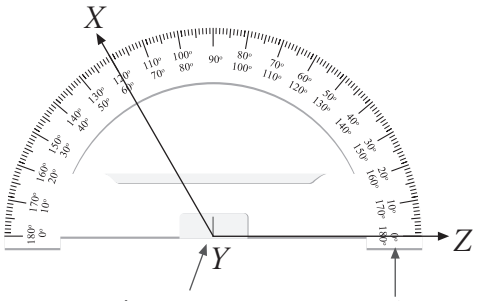
Using a protractor to measure angles

The amount of turn between each arm is measured in degrees with the aid of a protractor.

Measure the size of $\angle XYZ$

Step 1: Set up protractor to measure

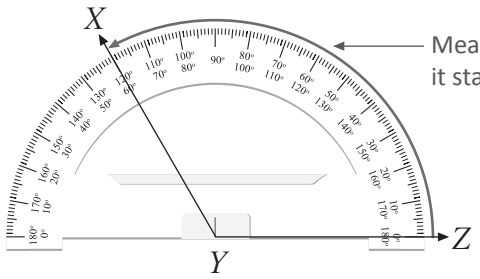
Remember: the vertex is the pointy bit



Place the centre marker on the protractor at the vertex

Line up one of the arms with a 0° mark

Step 2: Read the angle



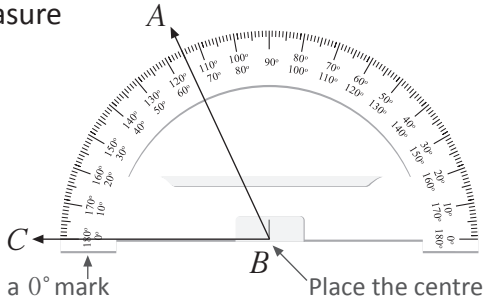
Measure with the outside scale as it starts with 0°

$\therefore \angle XYZ = 120^\circ$

The two scales on a protractor enable us to measure angles from either direction.

Measure the size of $\angle ABC$

Step 1: Set up protractor to measure

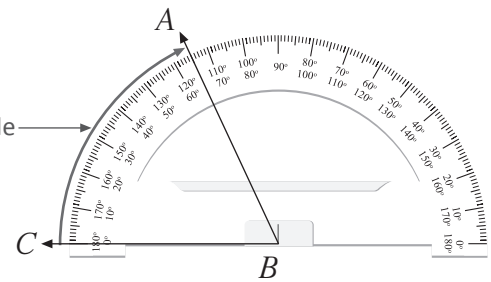


Line up one of the arms with a 0° mark

Place the centre marker on the protractor at the vertex

Step 2: Read the angle

Measure with the inside scale as it starts with 0°

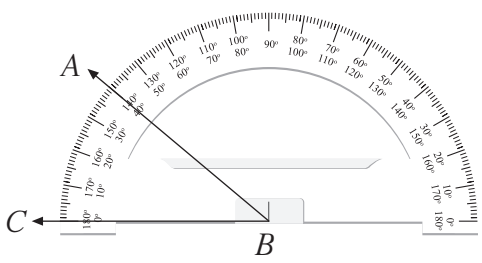


$\therefore \angle ABC = 65^\circ$

Using a protractor to measure angles

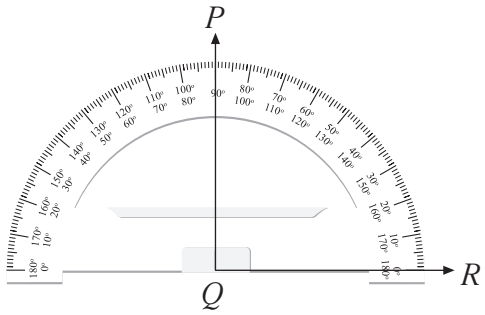
1 Write down the size (amount of turn in degrees) of these measured angles.

a



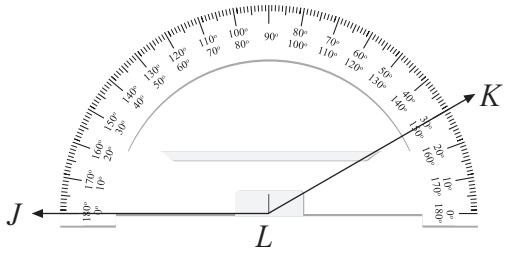
$\therefore \angle ABC =$ .....

b



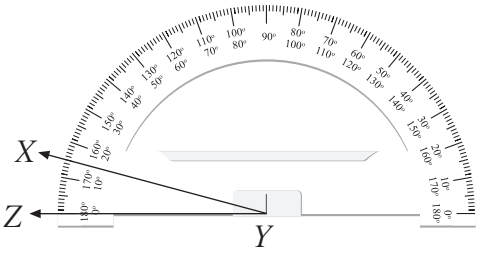
$\therefore \angle PQR =$ .....

c



$\therefore \angle JLK =$ .....

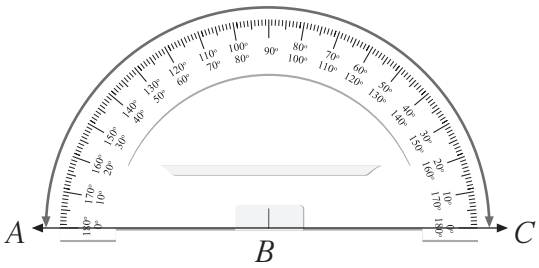
d



$\therefore \angle XYZ =$ .....

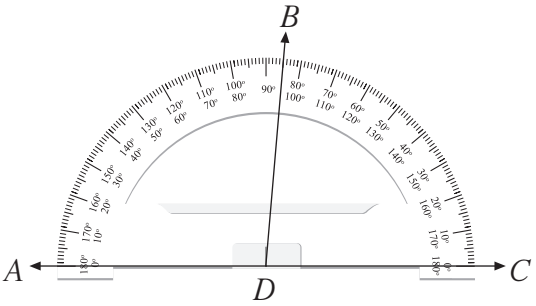
2 Write down the size of the angles indicated below each diagram.

a



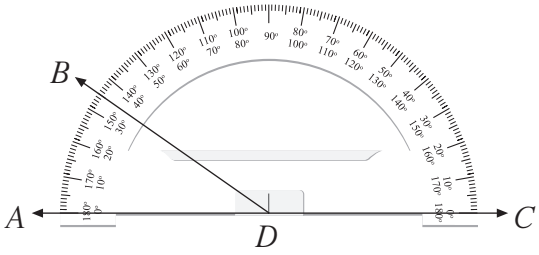
$\therefore \angle ABC =$ .....

b



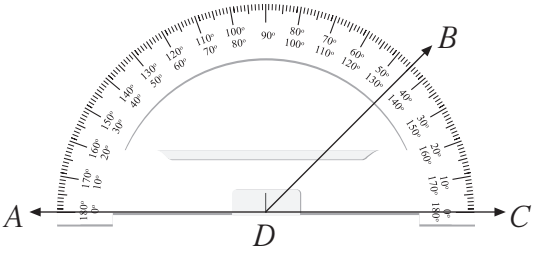
$\therefore \angle BDC =$ .....

c



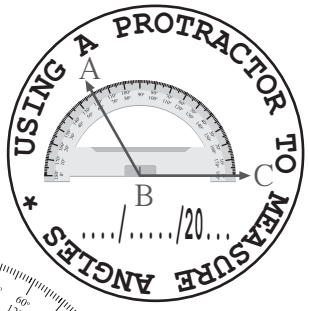
$\therefore \angle CDB =$ .....

d



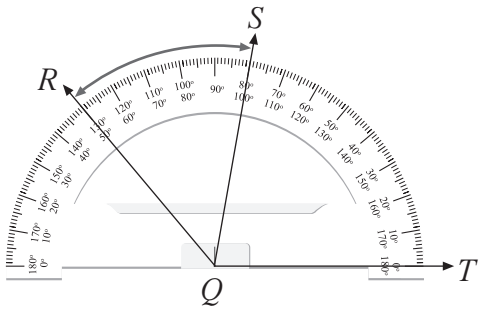
$\therefore \angle ADB =$ .....

Using a protractor to measure angles



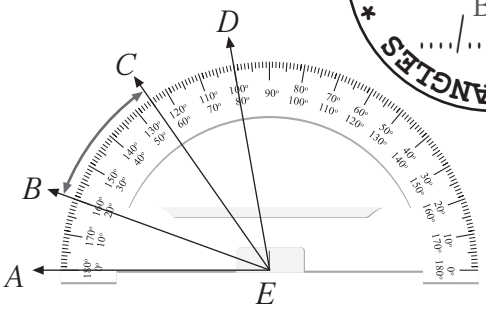
3 Try these trickier ones!

a



$\angle RQS =$ .....

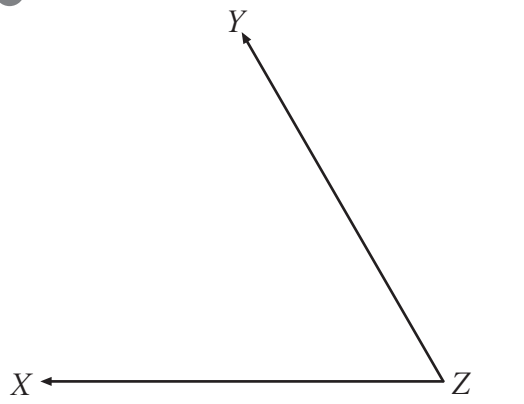
b



$\angle BEC =$ .....

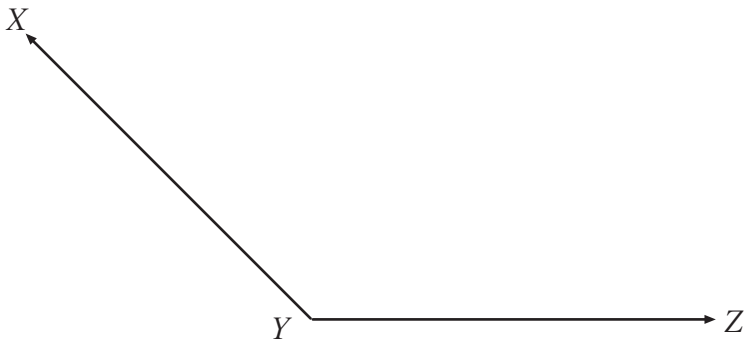
4 Use a protractor to measure the size (amount of turn in degrees) between the arms for these four angles:

a



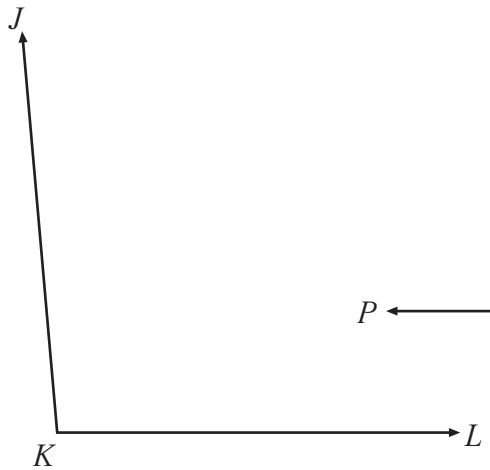
$\therefore \angle XZY =$ .....

b



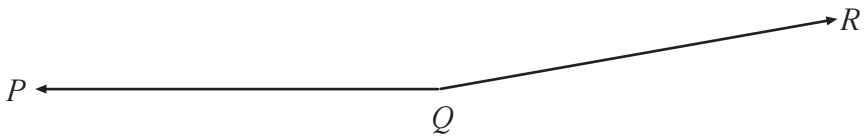
$\therefore \angle XYZ =$ .....

c



$\therefore \angle JKL =$ .....

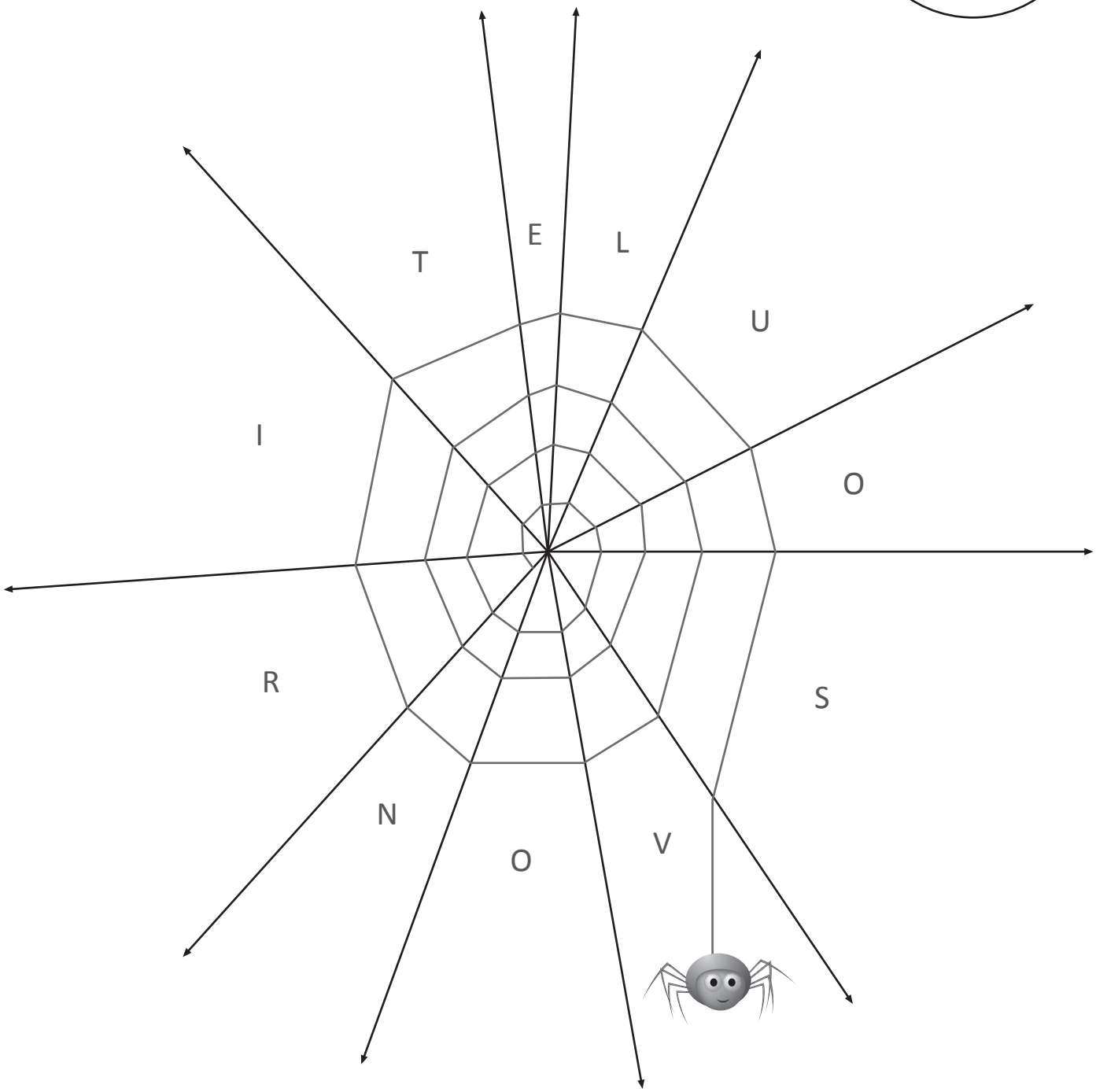
d



$\therefore \angle PQR =$ .....

Using a protractor to measure angles

5 Measure each acute angle between the straight supports on Bert the spider's web and match the letter with the correct size below.



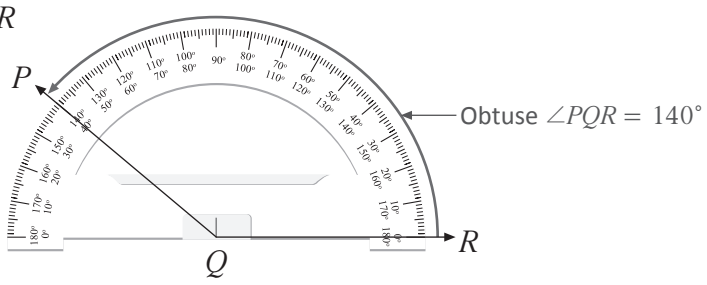
44°	10°	24°	30°	20°	40°	35°	52°	27°	22°	56°

Using a protractor to measure reflex angles

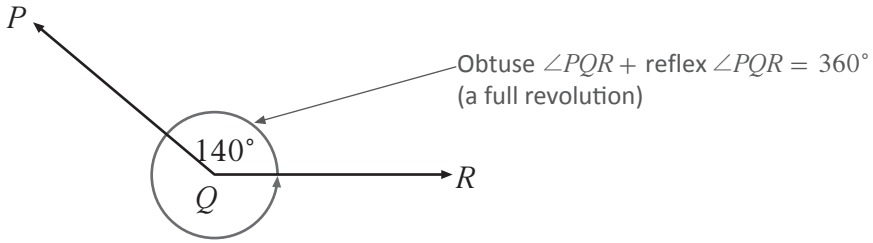
Most protractors only measure angles up to $180°$, so measure the angle you can and go from there.

Measure the size of reflex $\angle PQR$

Step 1: Measure the obtuse $\angle PQR$



Step 2: Subtract the size of the obtuse angle from $360°$



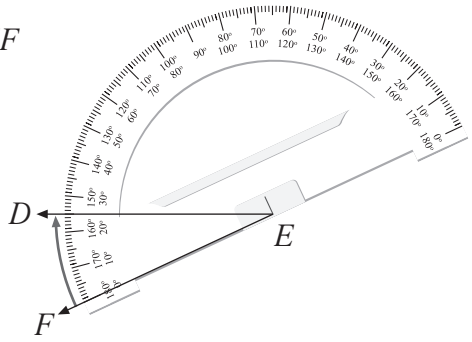
$\therefore$ Reflex $\angle PQR = 360° - 140°$

$= 220°$

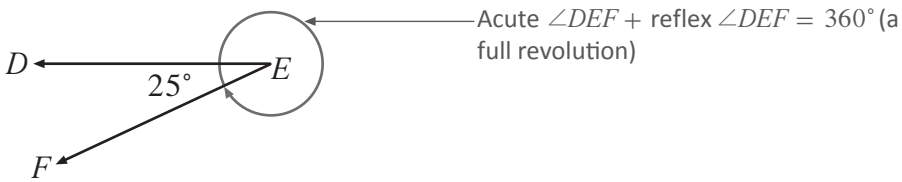
Here is another example with an acute angle.

Measure the size of reflex $\angle DEF$

Step 1: Measure the acute $\angle DEF$



Step 2: Subtract the size of the acute angle from $360°$



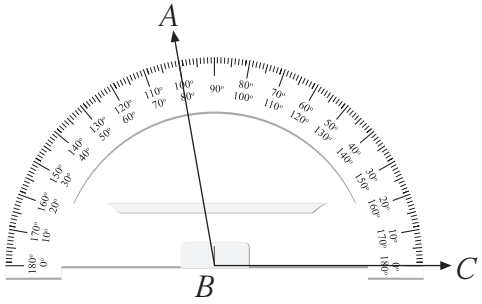
$\therefore$ Reflex $\angle DEF = 360° - 25°$

$= 335°$

Using a protractor to measure reflex angles

1 Calculate the size of these reflex angles.

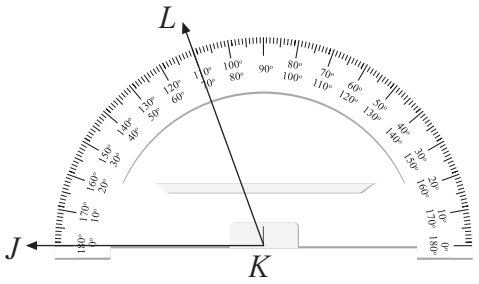
a



$\therefore$ Reflex $\angle ABC = 360° -$ ____

$=$ ____

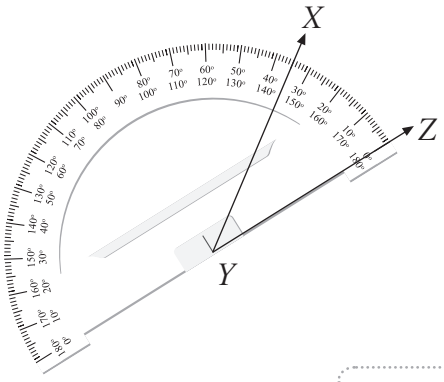
b



$\therefore$ Reflex $\angle JLK = 360° -$ ____

$=$ ____

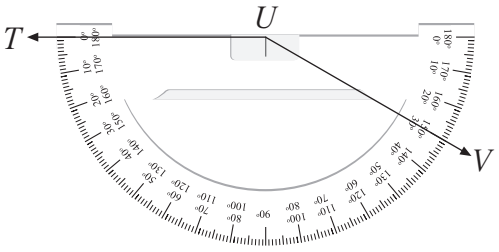
c



$\therefore$ Reflex $\angle XYZ = 360° -$ ____

$=$ ____

d

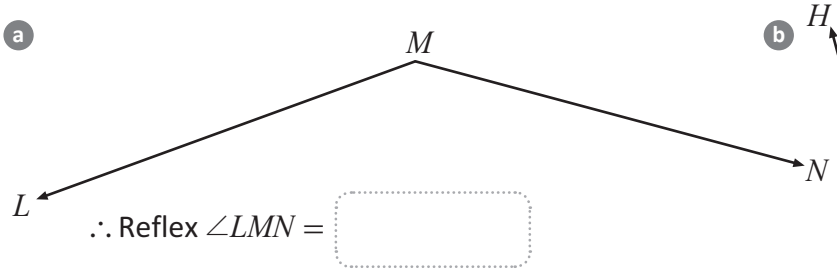


$\therefore$ Reflex $\angle TUV = 360° -$ ____

$=$ ____

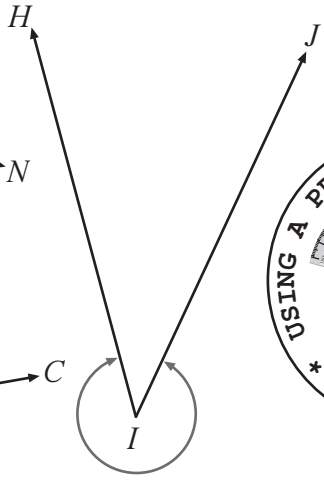
2 Measure and write down the size of the reflex angle for each of these:

a

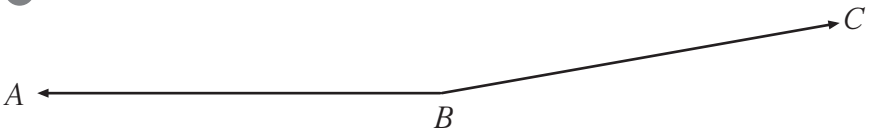


$\therefore$ Reflex $\angle LMN =$ ____

b

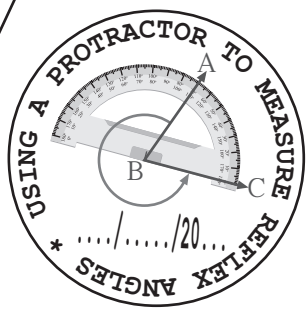


c



$\therefore$ Reflex $\angle ABC =$ ____

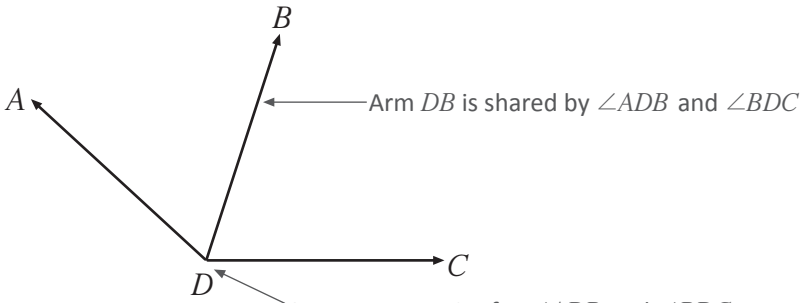
$\therefore$ Reflex $\angle HIJ =$ ____



Adjacent Angles

Angles that do **not** overlap and share an arm from the same vertex point are called adjacent angles.

Name the adjacent acute angles in this diagram



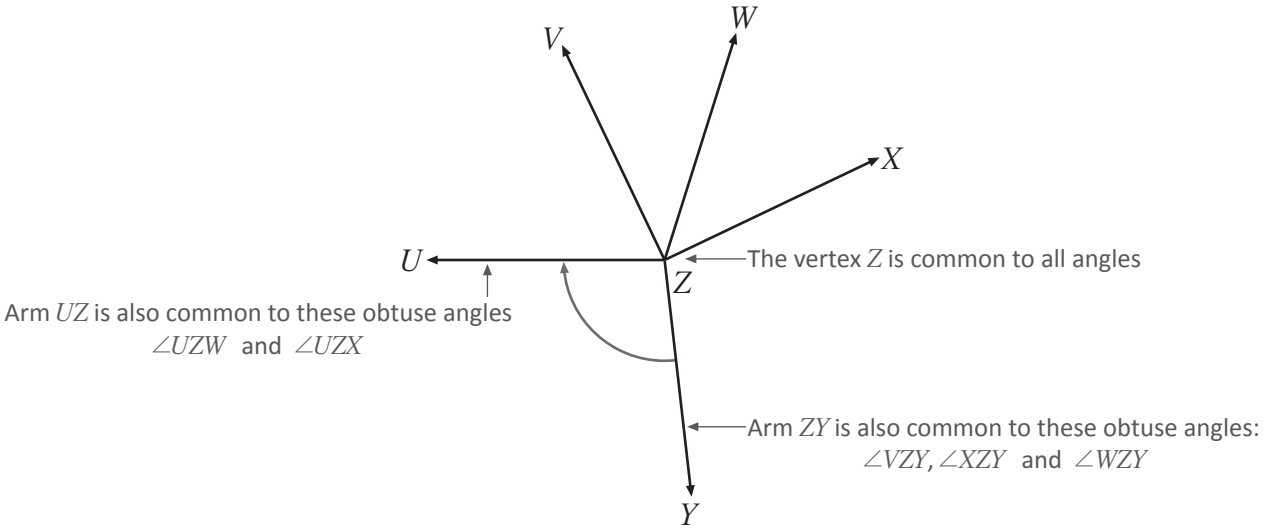
$\angle ADB$ is adjacent to $\angle BDC$

Adjacent simply means 'next to'

Here is the official way to say it: "The arm DB is common to both angles."
"The vertex D is common to both angles."

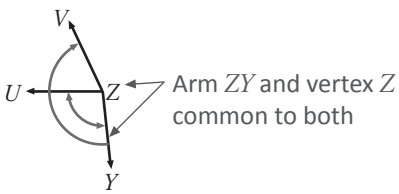
Here is an example where angles with a common arm and vertex overlap.

Name all the obtuse angles adjacent to $\angle UZY$

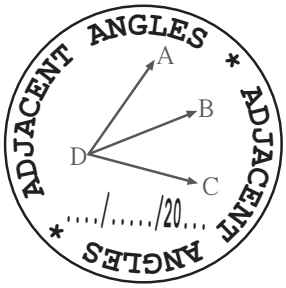


$\angle UZY$ is adjacent to the obtuse angles $\angle UZW, \angle UZX, \angle XZY$ and $\angle WZY$.

Be careful:
Eg: $\angle VZY$ and $\angle UZY$ share a common arm and vertex, but they are not adjacent because they overlap.

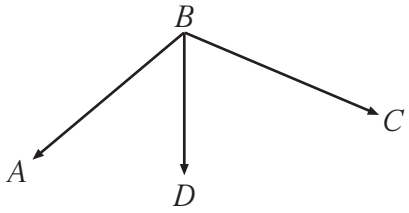


Adjacent angles



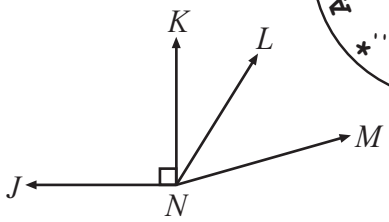
1 Name a pair of adjacent acute angles in each of these diagrams:

a



and

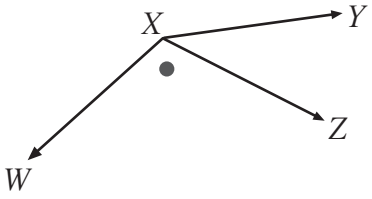
b



and

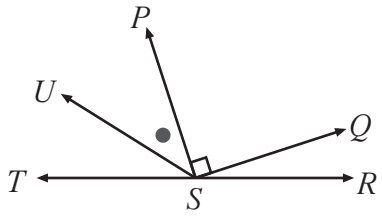
2 Name a **reflex** angle and an **acute** angle adjacent to these angles marked with a dot.

a



and

b



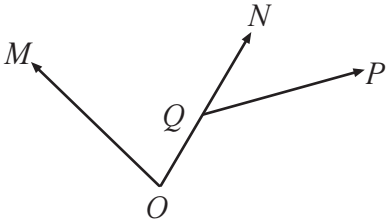
and

Remember to write the word reflex infront of the reflex angles

3 Draw an obtuse angle and label it $\angle PQR$. Draw an acute angle $\angle PQS$ adjacent to it.

4 Each of these angles share an arm. Explain why they are not adjacent to each other.

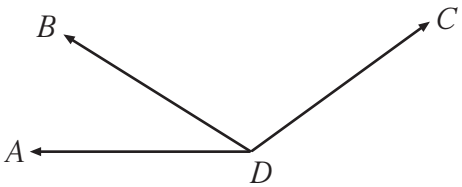
a



$\angle MON$ and $\angle NQP$

$\angle MON$ and $\angle NQP$ are not adjacent because:

b



$\angle ADB$ and $\angle ADC$

$\angle ADB$ and $\angle ADC$ are not adjacent because:

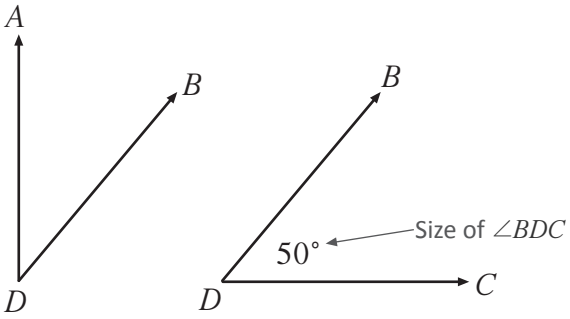
Complementary and supplementary angles

These special names are given to pairs of angles that add together to a total of 90° or 180°.

Complementary angles are a pair of angles that make a right-angle (90°) when put together.

Calculate the size of $\angle ADB$ if it is the complement of $\angle BDC$

Remember like this:
Complementary angles make a **C**orner.

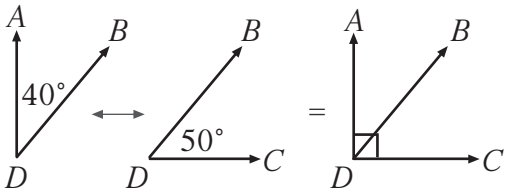


$\angle ADB + \angle BDC = 90^\circ$ $\angle ADB$ and $\angle BDC$ are complementary angles

$\therefore \angle ADB + 50^\circ = 90^\circ$

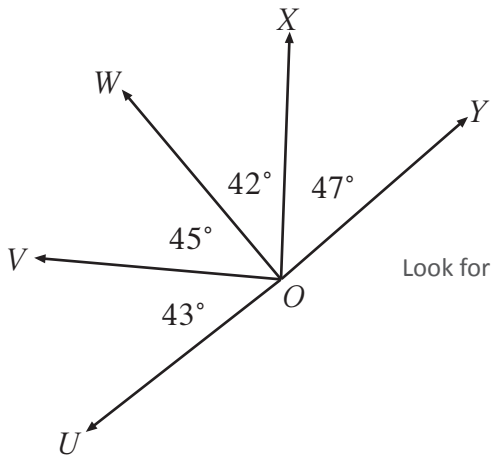
$\therefore \angle ADB = 40^\circ$

If you draw complementary angles adjacent to each other, you will make a right-angle!



Here is another example.

Name the pair of complementary angles in this diagram



Look for a pair of angles that add to 90°

$\angle UOV + \angle XOY = 43^\circ + 47^\circ$

$= 90^\circ$

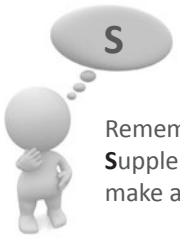
$\therefore \angle UOV$ and $\angle XOY$ are complementary angles

Supplementary angles are a pair of angles that make a straight-angle (180°) when put together.

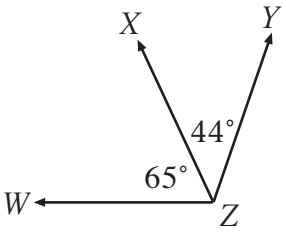
Calculate the supplement of 132°

The supplement of 132° is: $180^\circ - 132^\circ = 48^\circ$ Supplementary angles add to 180°

Calculate the size of an angle supplementary to $\angle WZY$



Remember like this:
Supplementary angles make a **S**traight angle.



$\angle WZY = \angle WZX + \angle XZY$ $\angle WZY$ is formed by two adjacent angles
$= 65^\circ + 44^\circ$
$= 109^\circ$

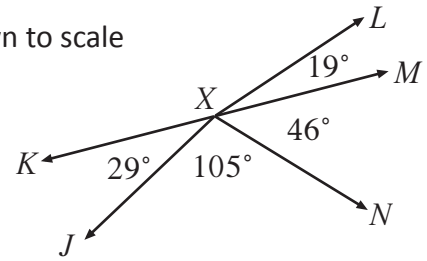
$\angle WZY +$ angle $= 180^\circ$
$\therefore 109^\circ +$ angle $= 180^\circ$
$\therefore$ angle $= 180^\circ - 109^\circ$
$\therefore$ angle $= 71^\circ$

$\therefore$ the size of the angle supplementary to $\angle WZY$ is 71°

Questions with many angles need a closer investigation.

Write down the pair of adjacent, supplementary angles from this diagram

Diagram not drawn to scale



$\angle MXJ = \angle MXN + \angle JXN$ The total size of other adjacent angles is sometimes needed
$\angle MXJ = 46^\circ + 105^\circ$
$\angle MXJ = 151^\circ$

$\angle MXJ + \angle JXK = 151^\circ + 29^\circ$
$= 180^\circ$

$\angle MXJ$ and $\angle JXK$ are supplementary angles

Complementary and supplementary angles

1 Calculate the complement of these angles:

a $30°$

b $80°$

c $46°$

d $11°$

e $23.5°$

f $18.3°$

2 Calculate the supplement of these angles:

a $100°$

b $90°$

c $165°$

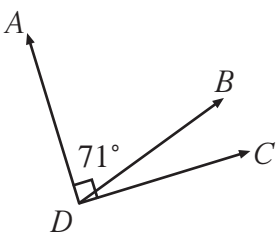
d $109°$

e $19\frac{1}{4}°$

f $121.3°$

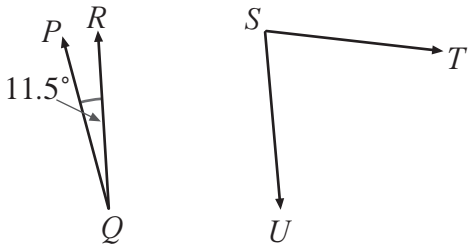
3 Calculate the size of the missing angle for these pairs of angles to be complementary:

a



$\angle BDC =$

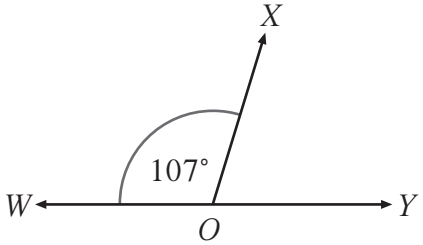
b



$\angle TSU =$

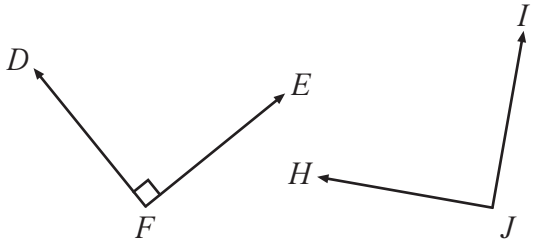
4 Calculate the size of the missing angle for these pair of angles to be supplementary:

a



$\angle XOY =$

b

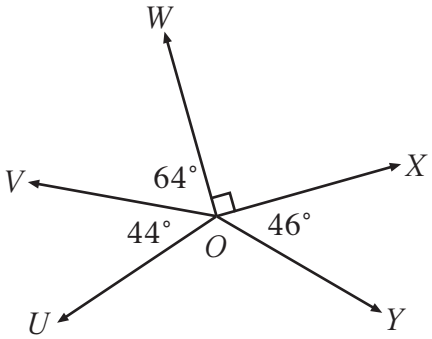


$\angle HJI =$

Complementary and supplementary angles

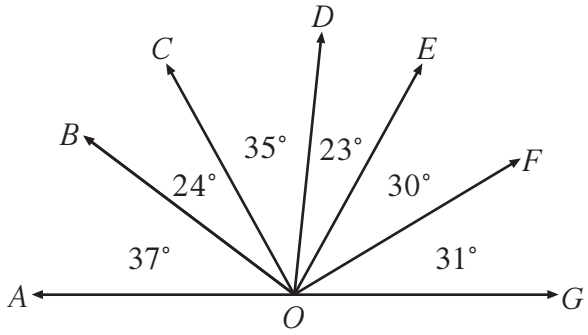
5 Name the pair of supplementary angles in this diagram:

Hint: what is the size of $\angle WOY$?



[ ] and [ ] are supplementary angles

6 Name the **angle** complementary to the ones given below:

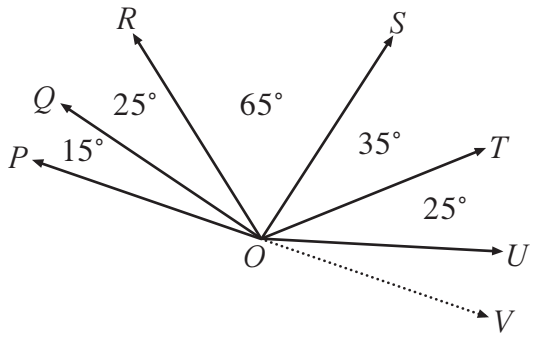


a $\angle DOF$ and [ ]

b $\angle FOG$ and [ ]

It's combo time!

7 a Name the pair of adjacent complementary angles in this diagram:



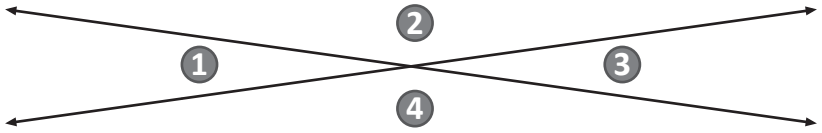
[ ] and [ ]

b If $\angle UOV$ is drawn adjacent to $\angle TOU$ as shown, what size must it be to make $\angle POV$ a straight angle? Hint: the angles must all add to $180°$

$\angle UOV =$ [ ]

Vertically opposite angles

When two straight lines cross each other, four angles are created.

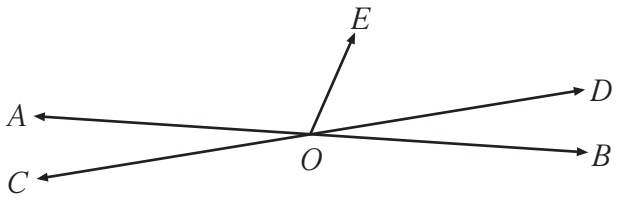


If you measured each of these angles with your protractor, you will discover that:

Angle 1 = Angle 3
and
Angle 2 = Angle 4

In Mathematics we refer to these equal angle pairs as vertically opposite angles.

Name the pairs of vertically opposite angles in this diagram:



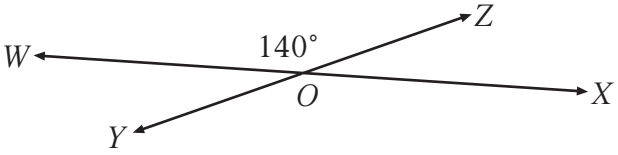
AB and CD are two straight lines crossing each other at O

$\therefore$ 1st pair of vertically opposite angles are: $\angle AOC$ and $\angle BOD$

$\therefore$ 2nd pair of vertically opposite angles are: $\angle AOD$ and $\angle BOC$

Adjacent angles formed by the intersection of two straight lines are supplementary.

Calculate the size of these angles:



(i) $\angle XOY$

$\angle XOY = \angle WOZ$ Vertically opposite angles are the same size

$\therefore \angle XOY = 140°$

(ii) $\angle WOY$

$\angle WOY$ is adjacent to $\angle WOZ$

$\therefore \angle WOY + \angle WOZ = 180°$ Adjacent angles of intersecting lines are supplementary

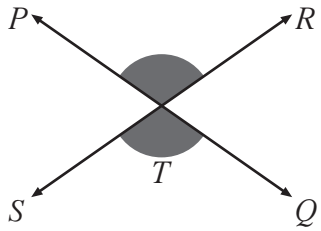
$\therefore \angle WOY + 140° = 180°$

$\therefore \angle WOY = 40°$

Vertically opposite angles

1 Name and shade all the pairs of vertically opposite angles below:

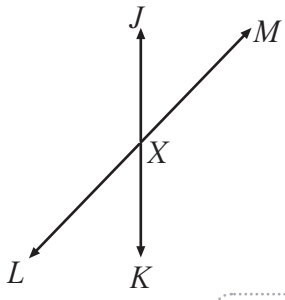
a



$\angle PTR$ and $\angle QTS$

and

b



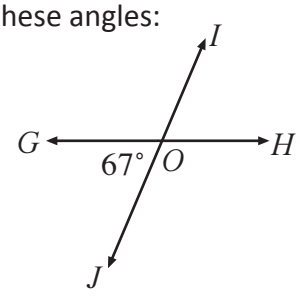
First pair: and

Second pair: and

2 Calculate the size of these angles:

a (i) $\angle IOH$

(ii) $\angle JOH$

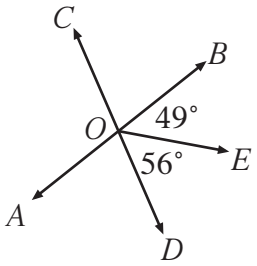


(i)

(ii)

b (i) $\angle COA$

(ii) $\angle AOD$

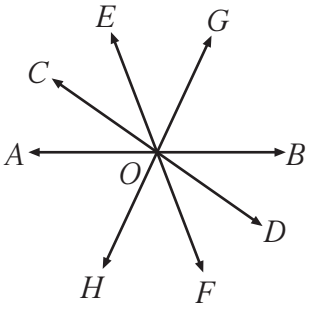


AB and *CD* are straight lines

(i)

(ii)

3 This diagram is made up of four straight lines *AB*, *CD*, *EF* and *GH* intersecting at the same point *O*. Name ten different pairs of vertically opposite angles.



(i) and (ii) and

(iii) and (iv) and

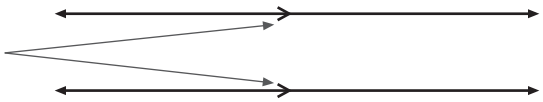
(v) and (vi) and

(vii) and (viii) and

(ix) and (x) and

Parallel lines

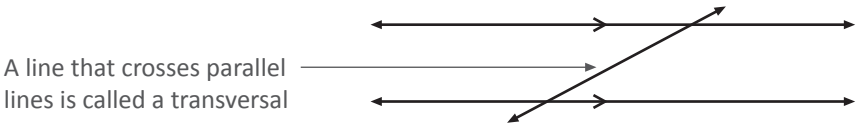
Parallel lines never cross each other, so on their own they never form an angle.



Parallel lines are named like this: $AB \parallel CD$

This symbol means 'is parallel to'

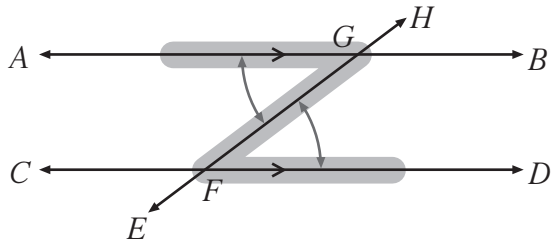
We get angles formed when another line that is not parallel crosses them.



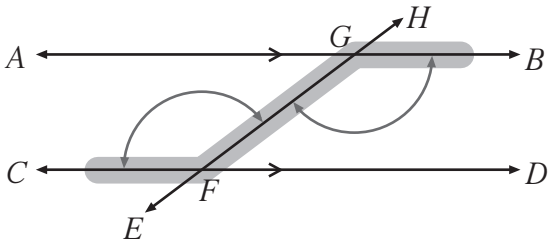
The opposite of parallel is 'perpendicular'. The symbol for this is $\perp$. It means the lines cross at 90° to each other

Alternate angles (Z-angles)

Angles that are on **alternate** sides of the transversal inside a pair of parallel lines are the same size.



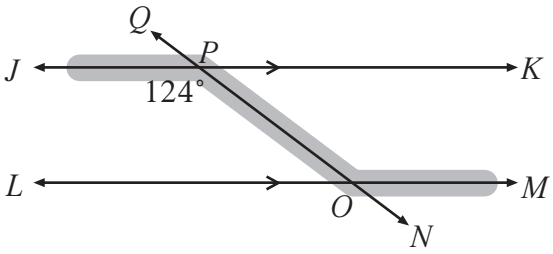
$\angle AGF = \angle DFG$

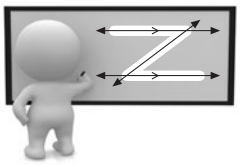


$\angle CFG = \angle BGF$

These are called **alternate angles**, and they form a 'zigzag' shape when highlighted. Let's call them 'Z-angles'.

Find the size of $\angle POM$





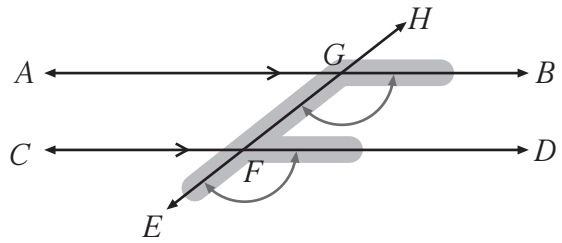
$\angle JPO = 124^\circ$

$\angle JPO = \angle POM$ Alternate angles in parallel lines, $JK \parallel LM$

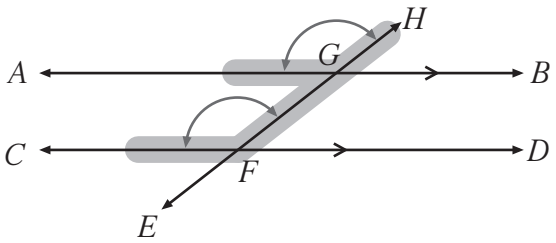
$\therefore \angle POM = 124^\circ$ Alternate angles in parallel lines are the same size

Corresponding angles (F-angles)

Angles that are in **corresponding** (matching) positions on a pair of parallel lines are the same size.



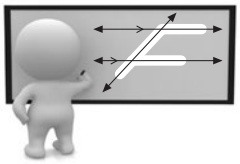
$\angle EFD = \angle FGB$

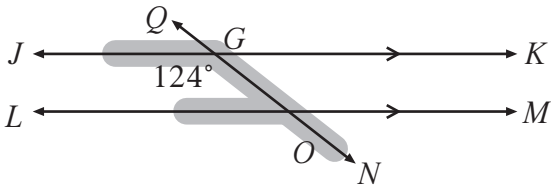


$\angle CFG = \angle AGH$

These are called corresponding angles and they form an 'F' shape when highlighted. Let's call them 'F-angles'.

Find the size of $\angle LON$





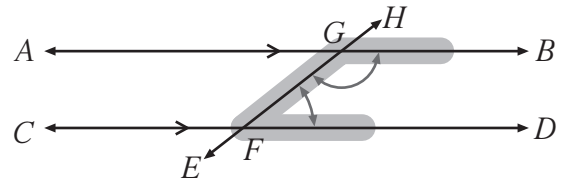
$\angle JPO = 124°$

$\angle JPO = \angle LON$ Corresponding angles in parallel lines, $JK \parallel LM$

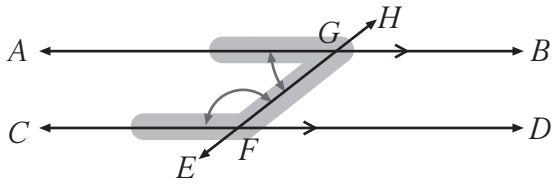
$\therefore \angle LON = 124°$ Corresponding angles in parallel lines are the same size

Cointerior angles (C-angles)

Angles on the same side of the transversal and inside a pair of parallel lines are supplementary.



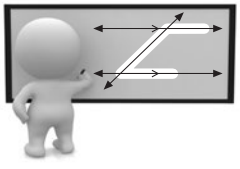
$\angle DFG + \angle FGB = 180°$

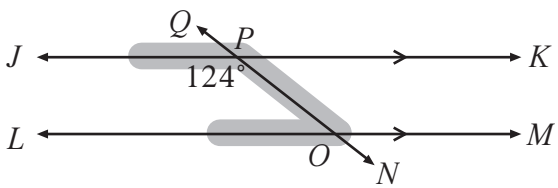


$\angle AGF + \angle CFG = 180°$

These are called cointerior angles and they form a 'C' shape when highlighted. Let's call these 'C-angles'.

Find the size of $\angle LOP$

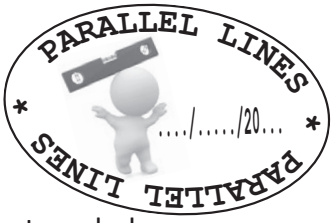




$\angle JPO = 124°$

$\angle JPO + \angle LOP = 180°$ Cointerior angles in parallel lines, $JK \parallel LM$

$\therefore \angle LOP = 56°$ Cointerior angles in parallel lines are supplementary

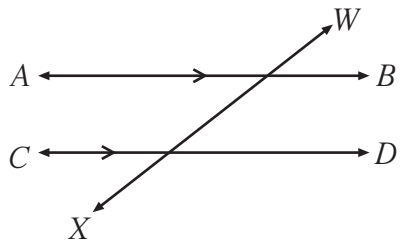


Parallel lines

1 For each of these diagrams: (i) Name the transversal.

(ii) Name the pair of parallel lines using the correct symbol.

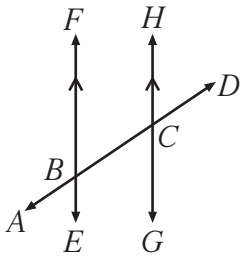
a



(i) _____

(ii) _____

b

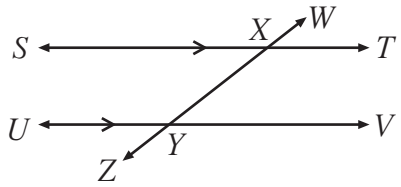


(i) _____

(ii) _____

2 Name all the pairs of alternate, corresponding and cointerior angles in these diagrams:

a



Alternate angles (Z-angles)

(i) _____ and _____

(ii) _____ and _____

Corresponding angles (F-angles)

There are four pairs of this type

(i) _____ and _____

(ii) _____ and _____

(iii) _____ and _____

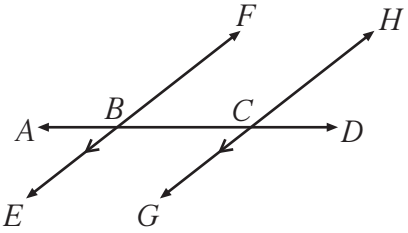
(iv) _____ and _____

Cointerior angles (C-angles)

(i) _____ and _____

(ii) _____ and _____

b



Alternate angles (Z-angles)

(i) _____ and _____

(ii) _____ and _____

Corresponding angles (F-angles)

There are four pairs of this type

(i) _____ and _____

(ii) _____ and _____

(iii) _____ and _____

(iv) _____ and _____

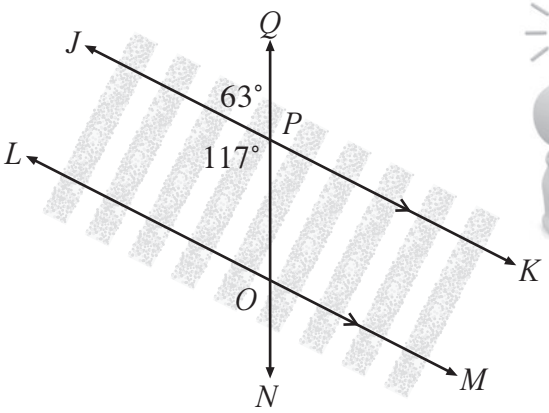
Cointerior angles (C-angles)

(i) _____ and _____

(ii) _____ and _____

Parallel lines

3 A straight cable QN, runs underneath a railway track as shown. Use the Z-angles and F-angles properties to complete the table with all the other angles that are the same size as the two given.

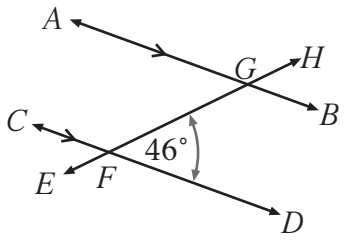


Remember you can look for vertically opposite angles too as they are also equal.

63°	117°
$\angle JPQ$	$\angle JPO$

4 Find the size of each of these angles and include one of the properties below you used to find them:
Properties: alternate , corresponding , cointerior , vertically opposite , straight

a



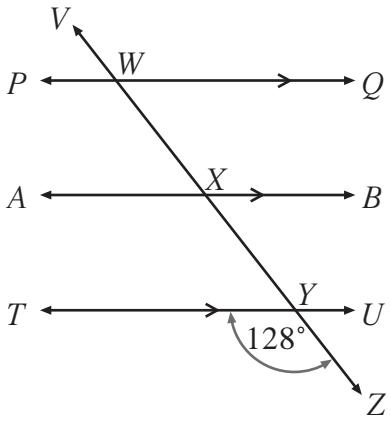
$\angle BGH =$ ____ Property used: Corresponding angles

$\angle AGF =$ ____ Property used: ____ angles

$\angle CFE =$ ____ Property used: ____ angles

$\angle BGF =$ ____ Property used: ____ angles

b Try this one with 3 parallel lines! (psst! You will need to use more than one property)



$\angle WXB =$ 128°

Property used:
Corresponding angle to $\angle TYZ$, $(\angle AXY) = 128°$
$\angle WXB$ is vertically opposite to $\angle AXY = 128°$

$\angle AXW =$ ____

Property used:

$\angle BXY =$ ____

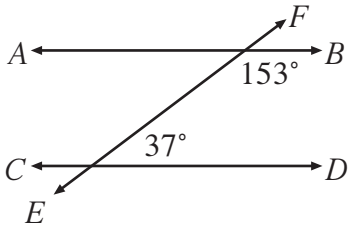
Property used:

Parallel lines

Since the rules for Z-angles, F-angles and C-angles only work when lines are parallel, you can use them to find out whether a pair of lines are parallel or not!

5 For each of these: (i) circle 'parallel' or 'not parallel' for the lines drawn
(ii) write a reason why you circled the one you did!

a



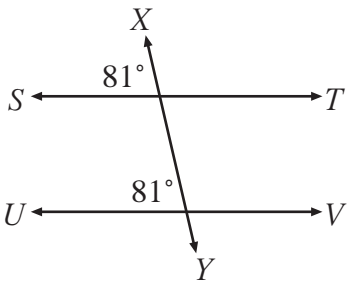
(i) The line AB and CD are:

PARALLEL NOT PARALLEL

(ii) Reason:

The cointerior angles do not add up to 180°
or
The cointerior angles are not supplementary

b

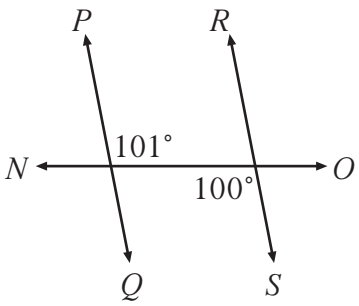


(i) The line ST and UV are:

PARALLEL NOT PARALLEL

(ii) Reason:

c

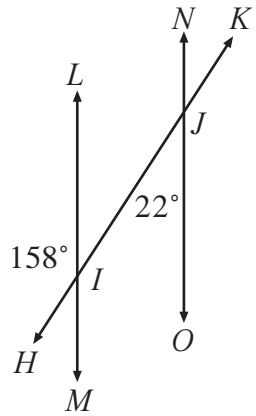


(i) The line PQ and RS are:

PARALLEL NOT PARALLEL

(ii) Reason:

d



(i) The line LM and NO are:

PARALLEL NOT PARALLEL

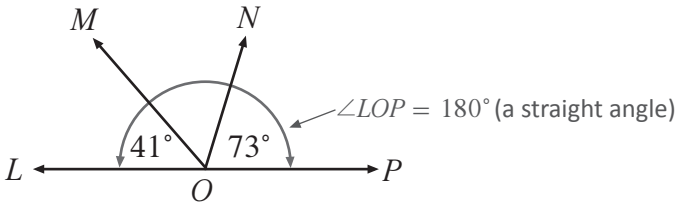
(ii) Reason: (write all the properties used here)

Angle sums

The size of special angle types learnt earlier can be used to find unknown angles.

Angles that form a straight line add to $180°$.

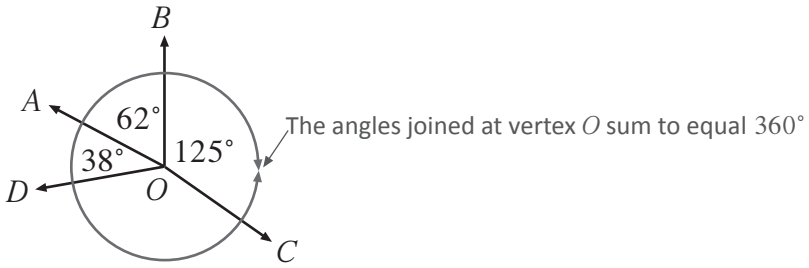
Calculate the size of $\angle MON$, if LP is a straight line



$$\angle LOM + \angle PON + \angle MON = \angle LOP = 180°$$
$$\therefore 41° + 73° + \angle MON = 180°$$
$$\therefore \angle MON = 180° - 41° - 73°$$
$$\angle MON = 66°$$

Angles that are part of a full revolution are called 'angles at a point' and they add to $360°$.

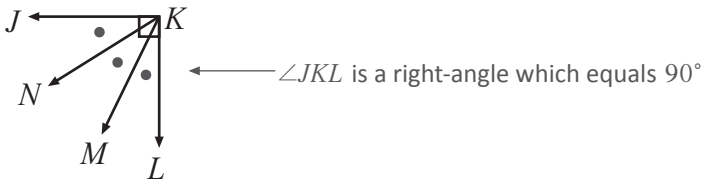
Calculate the size of $\angle COD$



$$\angle AOD + \angle AOB + \angle BOC + \angle COD = 360°$$
$$\therefore 38° + 62° + 125° + \angle COD = 360°$$
$$\therefore \angle COD = 360° - 38° - 62° - 125°$$
$$\angle COD = 135°$$

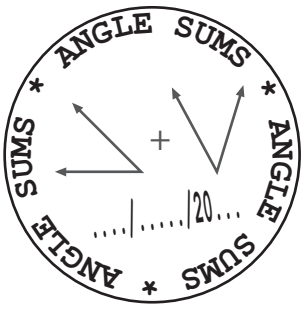
Here is a right-angle example.

Calculate the size of $\angle JKN, \angle MKN$ and $\angle LKM$ if they are all the same size



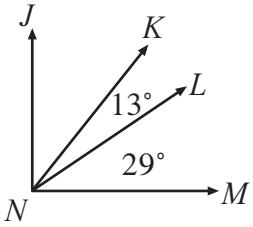
$$\angle JKN + \angle MKN + \angle LKM = 90°$$
$$\therefore \angle JKN, \angle MKN \text{ and } \angle LKM = 90° \div 3 \quad \div 3 \text{ as they are all the same size}$$
$$\therefore \angle JKN, \angle MKN \text{ and } \angle LKM = 30°$$

Angle sums



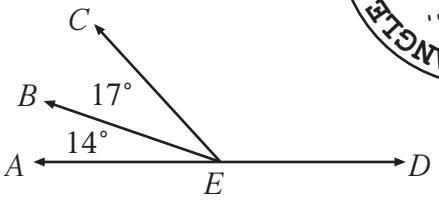
1 For each of these diagrams, calculate the size of the missing angle:

a



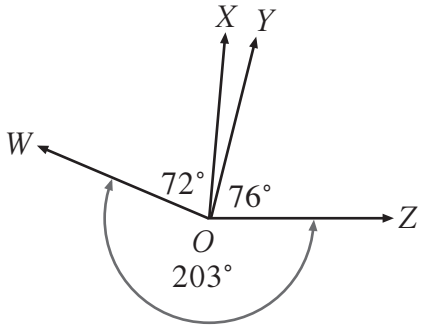
$\angle JNK =$

b



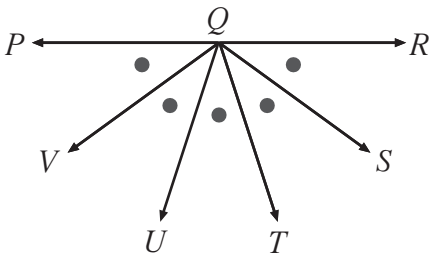
$\angle CED =$

c



$\angle XOY =$

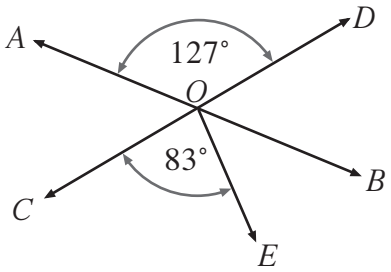
d Small dots can be used to show equal sized angles



Each angle =

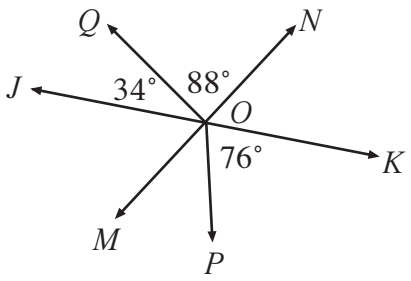
2 Vertically opposite angles can be used to help find the unknown angles for these.

a AB and CD are straight lines



$\angle BOE =$

b JK and MN are straight lines

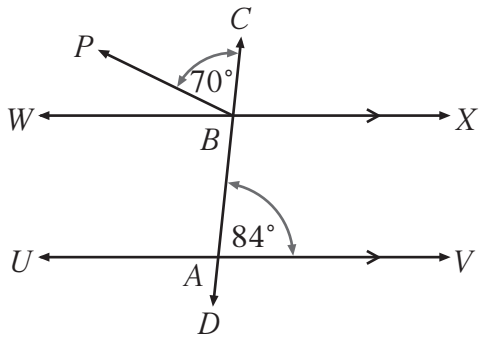


$\angle MOP =$

Angle sums

3 Use the parallel line angle properties to help find the size of these angles:

a



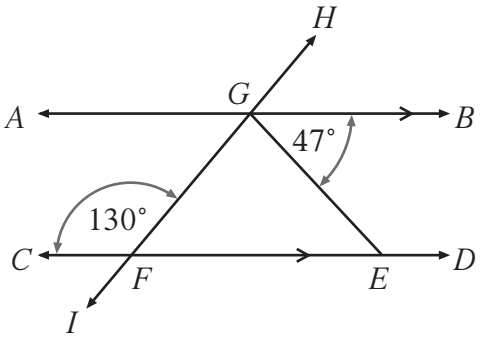
$\angle WBA =$

$\angle CBX =$

$\angle ABX =$

$\angle PBW =$

b



$\angle AGH =$

$\angle AGF =$

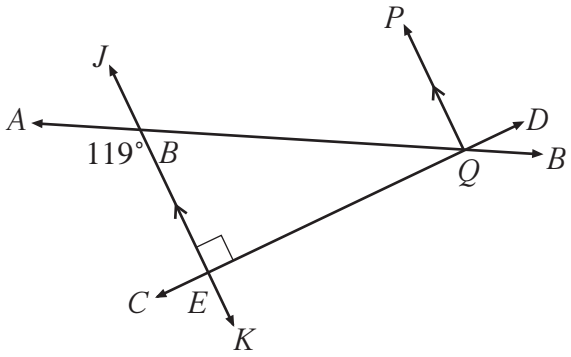
$\angle HGB =$

$\angle EGF =$

Combo Time!

4 Give these tricky ones a go! You have the skills now to use a few different angle properties for each one.

a JK and PQ are straight, parallel lines. AB and CD are straight lines.

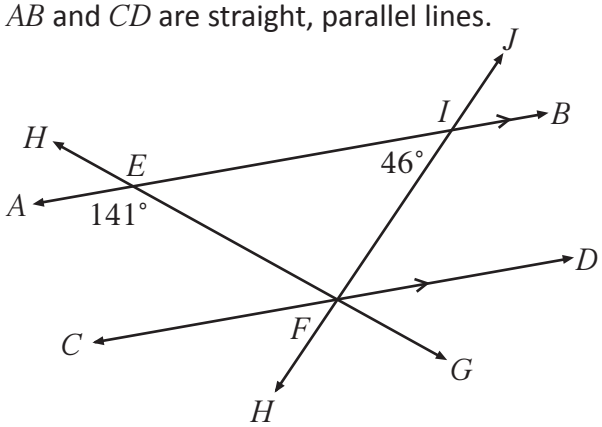


$\angle PQB =$

$\angle PQD =$

$\angle DQB =$

b AB and CD are straight, parallel lines.



$\angle DFG =$ Hint: find $\angle EFC$ first

$\angle EFI =$ Hint: find $\angle IFD$ first

Angle problems

Many real life problems can involve the application of the angle properties covered in this booklet.

This one uses the angle sum of a revolution.

Trinity is blindfolded and spun around in a children's party game. If it takes her $7\frac{1}{2}$ equal-sized steps to complete each circle, how many degrees does she turn with each step?

1 complete circle $=$ 1 revolution $=360^\circ$

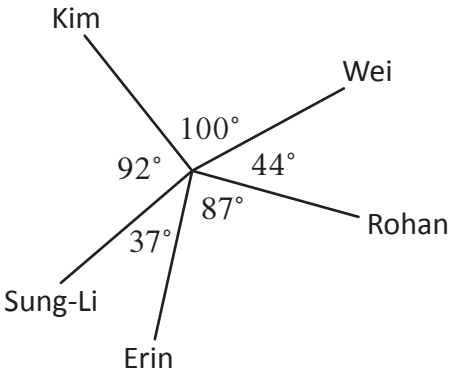
$\therefore$ Number of degrees turned with each step $= 360^\circ \div 7\frac{1}{2}$

$= 48^\circ$

$\therefore$ Trinity spins 48° with each step she takes. Always answer problems with a statement

Here is another problem.

Five people were holding lengths of rope at one end, they were all tied together at the other. They need to move around until the angles between each rope are the same. Explain how each person should move if Kim and Rohan must both remain still.



Calculate the size each angle needs to be.

$\therefore 360^\circ \div 5 = 72^\circ$ Angles at a point add to 360°

Kim and Rohan cannot move.

Wei moves $100^\circ - 72^\circ = 28^\circ$ counter-clockwise

$\therefore$ Angle between Kim and Wei $= 72^\circ$ ✓

Sung-Li moves $92^\circ - 72^\circ = 20^\circ$ clockwise

$\therefore$ Angle between Kim and Sung-Li $= 72^\circ$ ✓

Sung-Li's movements leave Erin $20^\circ + 37^\circ = 57^\circ$ away from her.

$\therefore$ Erin moves $72^\circ - 57^\circ = 15^\circ$ counter-clockwise

$\therefore$ Angle between Sung-Li and Erin $87^\circ - 15^\circ = 72^\circ$ ✓

Erin's movements take her 15° closer to Rohan

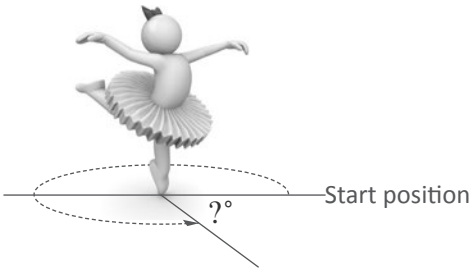
$\therefore$ Angle between Erin and Rohan $= 87^\circ - 15^\circ = 72^\circ$ ✓

$\therefore$ Angle between Wei and Rohan $= 72^\circ$ ✓ Only angle left over

Angle problems

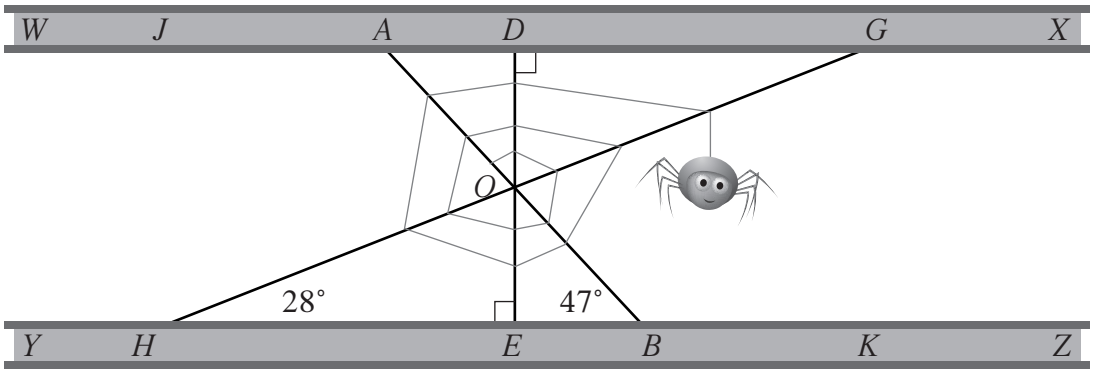
1 While performing a circular ballet move, Janet turned the first half easily and then with some extra effort, made it $\frac{5}{6}$ of the remaining way around.

a How many degrees was Janet away from completing the full circle?



b She immediately recovers and starts her second move facing where she had stopped. If she successfully turns another 180° in the same direction, how many degrees away from the start position is Janet now?

2 Bert is building another web, this time between two straight, parallel beams WX and YZ ($WX \parallel YZ$). His web has three straight supports: AB, DE and GH.



Bert wants to put in another straight support JK that passes through O, starting at J and finishing at K.

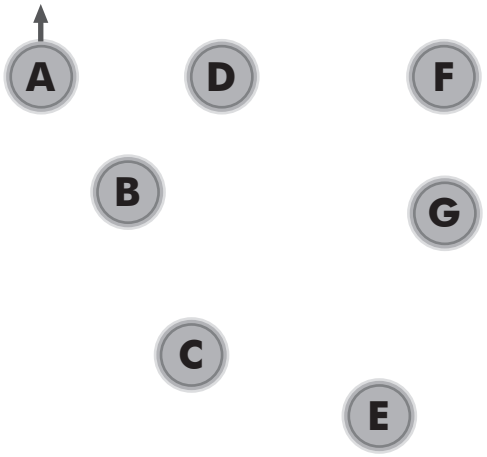
a Draw in the support JK that matches Bert's wishes.

b What is the size of $\angle OJA$ if all the acute angles $\angle OJA$, $\angle BAG$ and $\angle HGD$ are complementary?

Angle problems

3 A toy robot is programmed to move to all of the discs shown below.
It starts on disc A facing in the direction of the arrow.
When it reaches each disc, the robot remains facing the direction it was during the previous move.
Name the order of the discs it moves to if it follows these instructions in order:

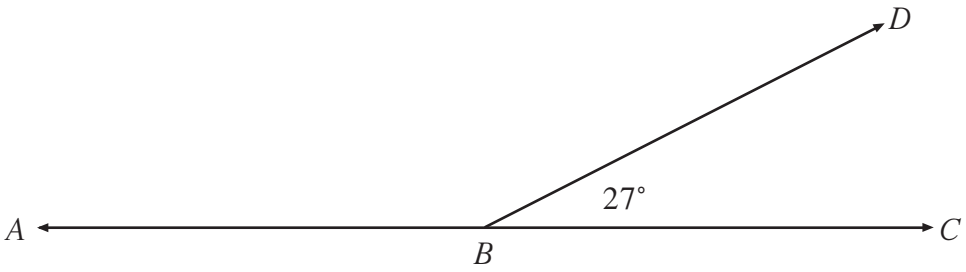
- Turn clockwise 90° ↻ and travel forward to the next disc.
- Complete a full revolution and then travel forward to the next disc.
- Turn counter-clockwise ↺ 200° then travel forward to the next disc.
- Turn clockwise 270° then travel forward to the next disc.
- Turn clockwise 80° then travel in reverse (backwards) to the next disc.
- Turn counter-clockwise an acute angle and travel forward to the last disc.



Disc order:

A → ○ → ○ → ○ → ○ → ○ → ○

4 As part of a treasure hunt, participants must complete puzzles to receive the name of the next destination. Here is the puzzle for one stop:



Step 1: If $\angle ABC$ is a straight angle, calculate the complement of $\angle DBC$.

Step 2: Add it to one of the angles formed when $\angle ABD$ is divided into nine equal sized angles.

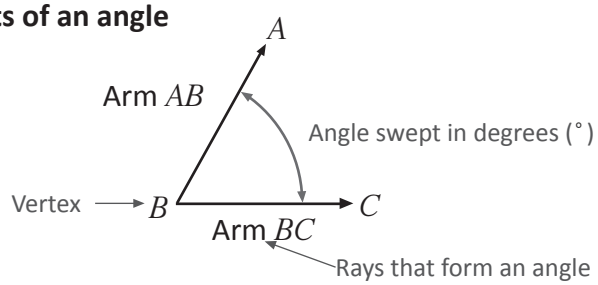
Step 3: Calculate the size of reflex $\angle ABD$.

Step 4: Subtract the value of step 2 from it and then add the supplement of $\angle DBC$ to the answer.

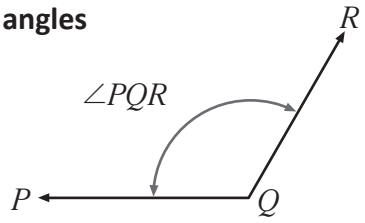
What answer will get you the name of the next destination?

Here is a summary of the important things to remember for angles

Parts of an angle



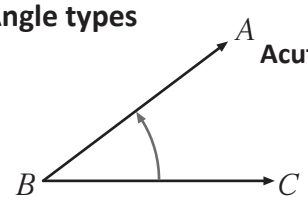
Naming angles



The letter at the vertex is **always** written in the middle

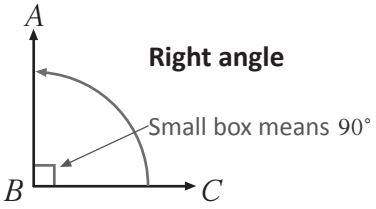
Angle types

Acute angle



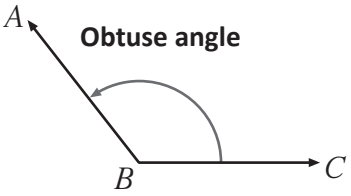
$0° < \angle ABC < 90°$

Right angle



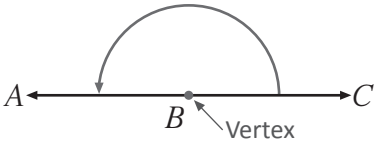
$\angle ABC = 90°$

Obtuse angle



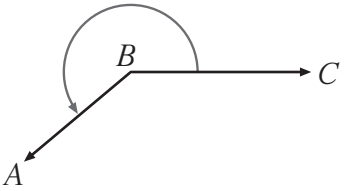
$90° < \angle ABC < 180°$

Straight angle



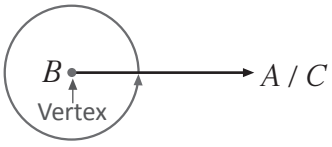
$\angle ABC = 180°$

Reflex angle



$180° <$ reflex $\angle ABC < 360°$

Full revolution or Full rotation



$\angle ABC = 360°$

Complementary and supplementary angles

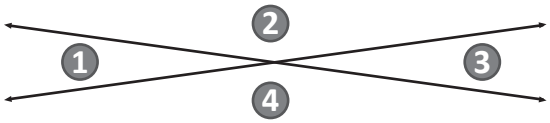
Complementary: a pair of angles whose sum $= 90°$

Together they form a right-angle

Supplementary: a pair of angles whose sum $= 180°$

Together they form a straight-angle

Vertically opposite angles



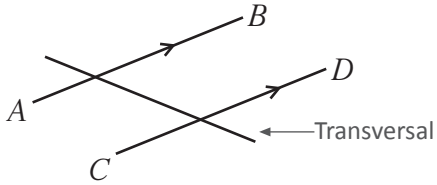
Angle 1 = Angle 3

and

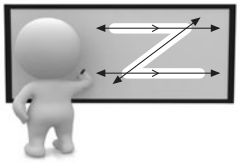
Angle 2 = Angle 4

Parallel lines

- $AB \parallel CD$: means the line AB is parallel to the line CD
- Arrows indicate parallel lines

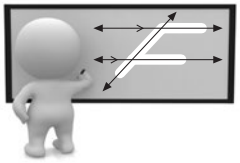


Alternate angles



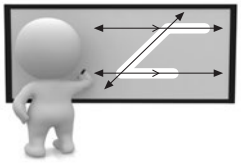
Equal on parallel lines

Corresponding angles



Equal on parallel lines

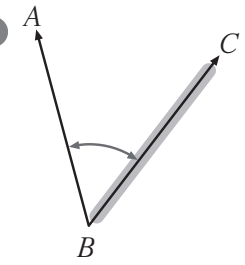
Cointerior angles



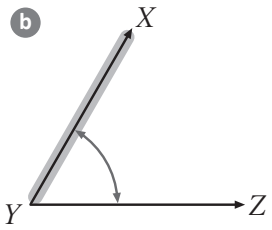
Supplementary on parallel lines

Parts of an angle

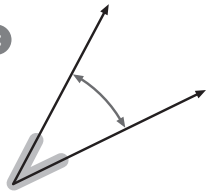
1. **a**



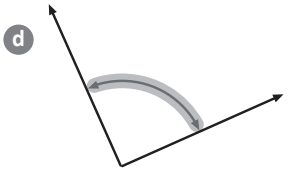
b



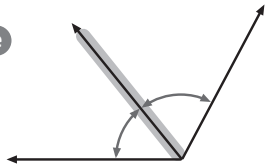
c



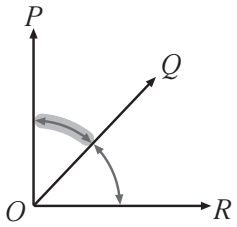
d



e



f



2. **a** The vertex **b** The angle swept

c The arm YZ **d** The common arm BD

e The arms SQ and RQ (also accept $\angle SQR$ or $\angle RQS$)

f The arms LO and NO (also accept $\angle LON$ or $\angle NOL$ or $\angle LOM$ or $\angle MON$)

Naming angles

1. **a** $\angle ACB$ or $\angle BCA$ or $\angle C$

b $\angle DFE$ or $\angle EFD$ or $\angle F$

c $\angle GJK$ or $\angle KJG$ or $\angle J$

2. **a** (i) $\angle XZW$ or $\angle WZX$ (ii) $\angle XZY$ or $\angle YZX$

b (i) $\angle ONP$ or $\angle PNO$ (ii) $\angle QNP$ or $\angle PNQ$

c (i) $\angle CFD$ or $\angle DFC$ (ii) $\angle AFB$ or $\angle BFA$

3. **a** Common arm is XZ **b** Common arm is PN

c No common arms

Naming angles

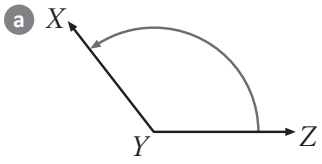
4. **a** $\angle LOM$ or $\angle MOL$ **b** $\angle ADC$ or $\angle CDA$

c $= \angle EDH$ or $\angle HDE$

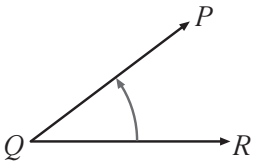
$= \angle FDH$ or $\angle HDF$

Angle types

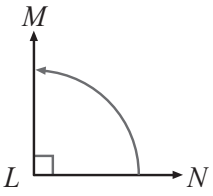
1. **a**



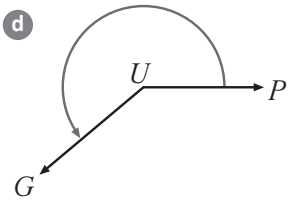
b



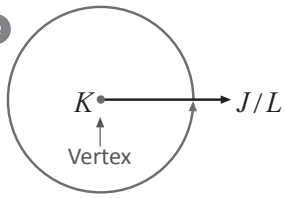
c



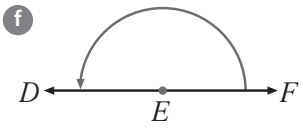
d



e



f



2.

Acute angle	Right angle	Obtuse angle
$\angle RQS$ or $\angle SQR$	$\angle RSQ$ or $\angle QSR$	$\angle PRW$ or $\angle WRP$
$\angle QPT$ or $\angle TPQ$	$\angle QST$ or $\angle TSQ$	$\angle PQS$ or $\angle SQP$
$\angle QRS$ or $\angle SRQ$	$\angle PTS$ or $\angle STP$	$\angle PTU$ or $\angle UTP$
$\angle RTU$ or $\angle UTR$	$\angle RTW$ or $\angle WTR$	$\angle TUW$ or $\angle WUT$
$\angle URS$ or $\angle SRU$	$\angle RUV$ or $\angle VUR$	$\angle TVU$ or $\angle UVT$
$\angle RUT$ or $\angle TUR$	$\angle VUW$ or $\angle WUV$	
$\angle UTV$ or $\angle VTU$		
$\angle TUV$ or $\angle VUT$		
$\angle UVW$ or $\angle WVU$		
$\angle UWV$ or $\angle VWU$		

Angle types

2.

Straight angle	Reflex angle
$\angle PQR$ or $\angle RQP$	Reflex ($\angle PRW$ or $\angle WRP$)
$\angle RST$ or $\angle TSR$	Reflex ($\angle RQS$ or $\angle SQR$)
$\angle PTV$ or $\angle VTP$	Reflex ($\angle QRS$ or $\angle SRQ$)
$\angle TVW$ or $\angle WVT$	Reflex ($\angle QST$ or $\angle TSQ$)
$\angle RUW$ or $\angle WUR$	Reflex ($\angle QSR$ or $\angle RSQ$)
	Reflex ($\angle STP$ or $\angle PTS$)
	Reflex ($\angle SRW$ or $\angle WRS$)
	Reflex ($\angle STU$ or $\angle UTS$)
	Reflex ($\angle UTV$ or $\angle VTU$)
	Reflex ($\angle STV$ or $\angle VTS$)
	Reflex ($\angle TUW$ or $\angle WUT$)
	Reflex ($\angle TUV$ or $\angle VUT$)
	Reflex ($\angle TVU$ or $\angle UVT$)
	Reflex ($\angle WVU$ or $\angle UVW$)
	Reflex ($\angle UWV$ or $\angle VWU$)
	Reflex ($\angle SRU$ or $\angle URS$)
	Reflex ($\angle RUT$ or $\angle TUR$)
	Reflex ($\angle PQS$ or $\angle SQP$)
	Reflex ($\angle QPT$ or $\angle TPQ$)

Getting 5 correct from this list is a good effort

Using a protractor to measure angles

1. (a) 40° (b) 90° (c) 150° (d) 15°

2. (a) 180° (b) 85° (c) 145° (d) 135°

3. (a) 50° (b) 35°

4. (a) 60° (b) 135° (c) 95° (d) 170°

5. REVOLUTIONS

Using a protractor to measure reflex angles

1. (a) 260° (b) 290° (c) 325° (d) 210°

2. (a) 215° (b) 320° (c) 190°

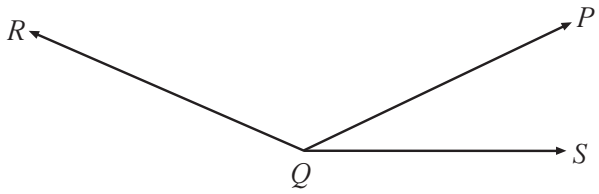
Adjacent angles

1. (a) $\angle ABD$ and $\angle CBD$ (b) $\angle KNL$ and $\angle LNM$

2. (a) reflex $\angle WXZ$ and $\angle YXZ$

(b) $\angle UST$, or reflex $\angle USR$, or reflex $\angle USQ$, or reflex $\angle USP$.

3. Here is one possible solution



4. (a) They share an arm, however that do not share a vertex, so they are not adjacent angles

(b) $\angle ADB$ forms part of $\angle ADC$, so since they overlap each other, they are not adjacent angles

Complementary and supplementary angles

1. (a) 60° (b) 10° (c) 44°
(d) 79° (e) 66.5° (f) 71.7°

2. (a) 80° (b) 90° (c) 15°
(d) 71° (e) $160\frac{3}{4}^\circ$ (f) 58.7°

3. (a) 19° (b) 78.5°

4. (a) 73° (b) 90°

5. $\angle WOY$ and $\angle UOV$

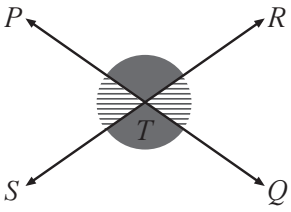
6. First pair: $\angle DOF$ and $\angle AOB$

Second pair: $\angle FOG$ and $\angle BOD$

7. (a) $\angle QOR$ and $\angle ROS$ (b) 15°

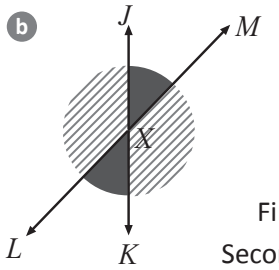
Vertically opposite angles

1. **a**



$\angle PTR$ and $\angle QTS$
$\angle PTS$ and $\angle RTQ$

b



First pair: $\angle JXM$ and $\angle KXL$
Second pair: $\angle JXL$ and $\angle KXM$

2. **a** (i) 67° (ii) 113°

b (i) 105° (ii) 113°

3.

$\angle AOC$ and $\angle BOD$	$\angle EOC$ and $\angle DOF$
$\angle EOG$ and $\angle HOF$	$\angle GOB$ and $\angle AOH$
$\angle AOE$ and $\angle BOF$	$\angle COG$ and $\angle DOH$
$\angle EOB$ and $\angle AOF$	$\angle COB$ and $\angle AOD$
$\angle AOG$ and $\angle BOH$	$\angle EOH$ and $\angle FOG$
$\angle COH$ and $\angle DOG$	$\angle COF$ and $\angle EOD$

Parallel lines

1. **a** (i) Transversal is the line WX

(ii) AB is prallel to CD, $AB \parallel CD$

b (i) Transversal is the line AD

(ii) EF is prallel to GH, $EF \parallel GH$

2. **a** Zangles (alternate angles):

$\angle SXY$ and $\angle VYX$ $\angle UYX$ and $\angle TXY$

Fangles (corresponding angles):

$\angle SXW$ and $\angle UYX$ $\angle SXY$ and $\angle UYZ$
$\angle WXT$ and $\angle XYV$ $\angle TXY$ and $\angle VYZ$

Cangles (cointerior angles):

$\angle SXY$ and $\angle UYX$ $\angle VYX$ and $\angle TXY$

Parallel lines

2. **b** Zangles (alternate angles):

$\angle FBC$ and $\angle GCB$ $\angle EBC$ and $\angle HCB$

Fangles (corresponding angles):

$\angle ABE$ and $\angle BCG$ $\angle EBC$ and $\angle GCD$
$\angle ABF$ and $\angle BCH$ $\angle FBC$ and $\angle HCD$

Cangles (cointerior angles):

$\angle EBC$ and $\angle GCB$ $\angle FBC$ and $\angle HCB$

3.

63°	117°
$\angle JPQ$	$\angle JPO$
$\angle KPO$	$\angle LON$
$\angle LOP$	$\angle MOP$
$\angle MON$	$\angle KPQ$

4. **a** 46° Corresponding angles.

46° Vertically opposite angles.

46° Vertically opposite angles
or
Corresponding angles.

134° Cointerior angles
or
Straight angle.

b 128° Corresponding angle
or
Vertically opposite angle.

52° Corresponding angle
or
Straight angle.

52° Vertically opposite angle
or
cointerior angle, so supplementary.

Parallel lines

5. **a** (i) NOT PARALLEL

(ii) The cointerior angles do not add up to 180°

or

The cointerior angles are not supplementary.

b (i) PARALLEL

(ii) The corresponding angles are the same size

c (i) NOT PARALLEL

(ii) The alternate angles are not equal to each other

d (i) PARALLEL

(ii) They are parallel because the cointerior angles are supplementary

Angle sums

1. **a** 48° **b** 149° **c** 9° **d** 36°

2. **a** 44° **b** 46°

3. **a** $\angle WBA = 84^\circ$
$\angle CBX = 84^\circ$
$\angle ABX = 96^\circ$
$\angle PBW = 26^\circ$

b $\angle AGH = 130^\circ$
$\angle AGF = 50^\circ$
$\angle HGB = 50^\circ$
$\angle EGF = 83^\circ$

4. **a** $\angle PQB = 119^\circ$
$\angle PQD = 90^\circ$
$\angle DQB = 29^\circ$

b $\angle DFG = 39^\circ$
$\angle EFI = 95^\circ$

Angle problems

1. **a** 30° **b** 150°

2. **b** 15°

3. A → D → F → B → C → G → E

4. 280°

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