Polygons

Curriculum Ready

Mathletics

www.mathletics.com

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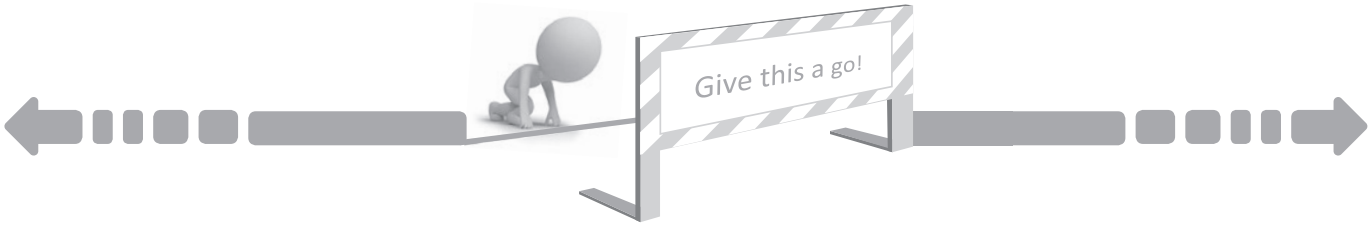
This booklet is about identifying and manipulating straight sided shapes using their unique properties

Many clever people contributed to the development of modern geometry including:

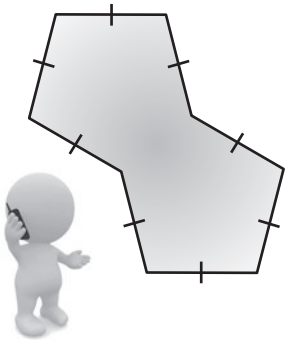
- Thales of Miletus (approx. $624-547$ BC)
- Pythagoras (approx. $569-475$ BC)
- Euclid of Alexandria (approx. $325-265$ BC) (often referred to as the "Father of modern geometry')
- Archimedes of Syracus (approx $287-202$ BC)
- Apollonius of Perga (approx. $261-190$ BC)

After an attack on the city of Alexandria, many of the works of these mathematicians were lost. Look up these people sometime and read about their contribution to this subject.

New discoveries in geometry are still being made with the advent of computers, in particular fractal geometry. The most famous of these being Benoit Mandelbrot Fractal pattern.



Q Write down how you would describe this shape over the phone to a friend who had to draw it accurately. Try it with a friend/family member and see if they draw this shape from your description.

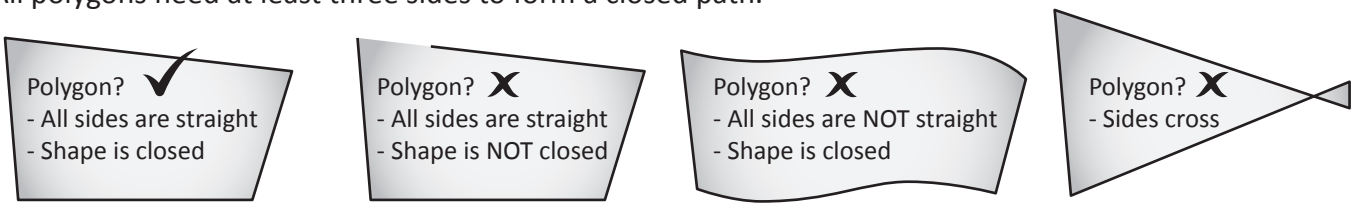


Work through the book for a great way to do this

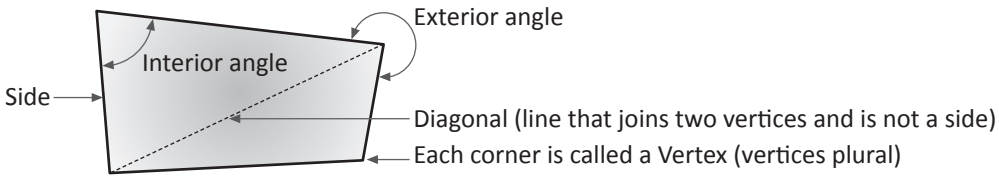
Polygons

Polygons are just any closed shape with straight lines which don't cross. Like a square or triangle.

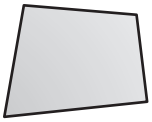
All polygons need at least three sides to form a closed path.



Parts of a polygon:



There are many basic types of polygons. Here are the ones we will be looking at in this booklet:

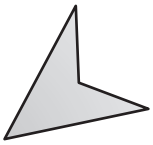


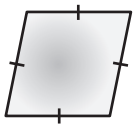
Convex polygon

All interior angles are $< 180°$

Concave polygon

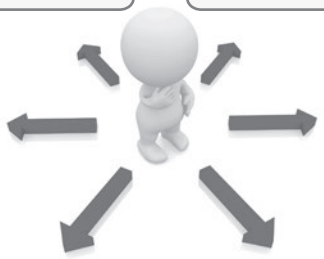
Has an interior angle $> 180°$





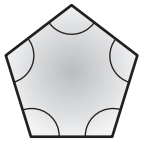
Equilateral polygon

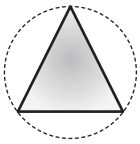
All sides are the same length



Equiangular polygon

All interior angles are equal



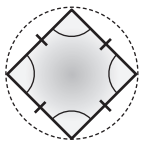


Cyclic polygon

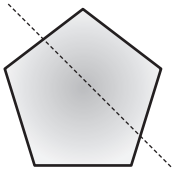
All vertices/corner points lie on the edge (circumference) of the same circle.

Regular polygon

All interior angles are equal
All sides are the same length
They are cyclic polygons

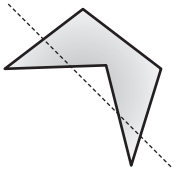


Here is another difference between convex and concave polygons.



Convex

A straight line drawn through the polygon can only cross a maximum of 2 sides



Concave

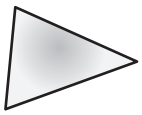
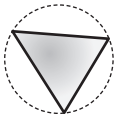
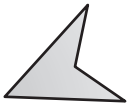
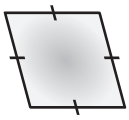
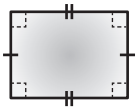
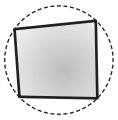
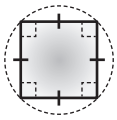
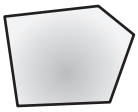
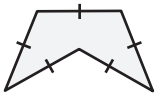
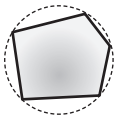
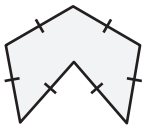
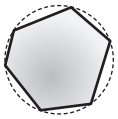
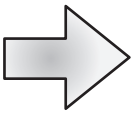
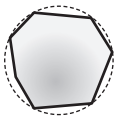
A straight line drawn through the polygon can cross more than two sides.

Polygons

Any polygon can be named using Greek prefixes matching the number of straight sides it has.

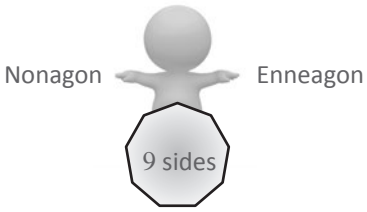
III = Trio	IIII = Tetra	卌 = Penta	卌 I = Hexa
卌 II = Hepta	卌 III = Octa	卌 IIII = Nona	卌 卌 = Deca
卌 卌 I = Hendeca	卌 卌 II = Dodeca	卌 卌 III = Trideca	卌 卌 IIII = Tetradeca

Polygon naming and classification chart

Sides	Name	Convex	Concave	Equilateral	Equiangular	Cyclic	Regular
3	Triangle (Trigon)		N/A				
4	Quadrilateral (Tetragon)						
5	Pentagon						
6	Hexagon						
7	Heptagon						
8	Octagon						

Here are some more polygon names.

Sides	Polygon name	Sides	Polygon name
9	Nonagon	19	Enneadecagon
10	Decagon	20	Icosagon
11	Hendecagon	30	Tricontagon
12	Dodecagon	40	Tetracontagon
13	Tridecagon	50	Pentacontagon
14	Tetradecagon	60	Hexacontagon
15	Pentadecagon	70	Heptacontagon
16	Hexadecagon	80	Octacontagon
17	Heptadecagon	90	Enneacontagon
18	Octadecagon	100	Hectogon



Many of these polygons have more than one name.

Look them up sometime!

Polygons

1 Identify which of these shapes are polygons or not.

a

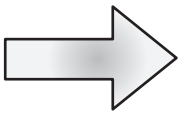
- ☐ Polygon
- ☐ Not a polygon

b

- ☐ Polygon
- ☐ Not a polygon

c

- ☐ Polygon
- ☐ Not a polygon

d 

- ☐ Polygon
- ☐ Not a polygon

e

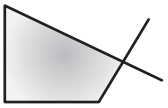
- ☐ Polygon
- ☐ Not a polygon

f

- ☐ Polygon
- ☐ Not a polygon

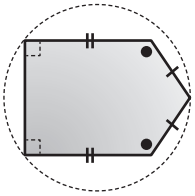
g

- ☐ Polygon
- ☐ Not a polygon

h 

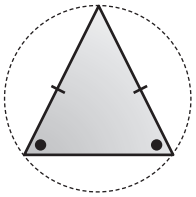
- ☐ Polygon
- ☐ Not a polygon

2 Tick all the properties that each of these polygons have and then name the shape:

a 

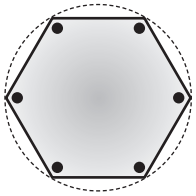
- ☐ Convex
- ☐ Concave
- ☐ Equilateral
- ☐ Equiangular
- ☐ Cyclic
- ☐ Regular

☐

b 

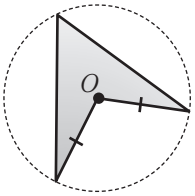
- ☐ Convex
- ☐ Concave
- ☐ Equilateral
- ☐ Equiangular
- ☐ Cyclic
- ☐ Regular

☐

c 

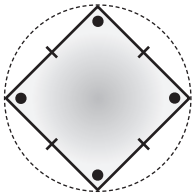
- ☐ Convex
- ☐ Concave
- ☐ Equilateral
- ☐ Equiangular
- ☐ Cyclic
- ☐ Regular

☐

d 

- ☐ Convex
- ☐ Concave
- ☐ Equilateral
- ☐ Equiangular
- ☐ Cyclic
- ☐ Regular

☐

e 

- ☐ Convex
- ☐ Concave
- ☐ Equilateral
- ☐ Equiangular
- ☐ Cyclic
- ☐ Regular

☐

f

- ☐ Convex
- ☐ Concave
- ☐ Equilateral
- ☐ Equiangular
- ☐ Cyclic
- ☐ Regular

☐

3 Draw and label:

a A regular tetragon.

b A concave nonagon.

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Polygons

POLYGONS * POLYGONS * POLYGONS * ...../...../20....

4 Draw and label:

a A convex, equilateral hexagon.

b A convex, cyclic tetragon which is not equilateral.

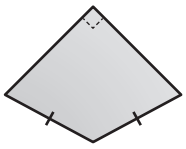
c An equiangular, pentagon which is not equilateral.

d A concave, equilateral heptagon with two reflex angles ($180° < \text{angle} < 360°$).

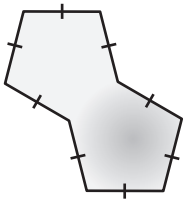
5 Explain why it is not possible to draw a cyclic, equilateral, concave octagon.

6 How would you describe these polygons to someone drawing them in another room?

a



b

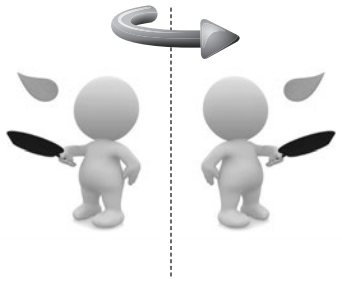


Transformations

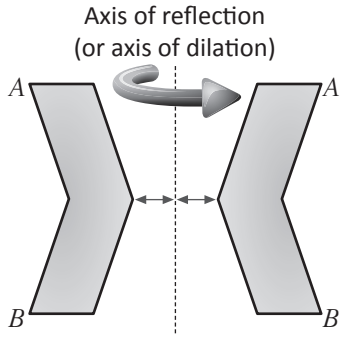
Transformations are all about re-positioning shapes without changing any of their dimensions.

There are three main types:

Reflections (Flip) Reflecting an object about a fixed line called the axis of reflection.

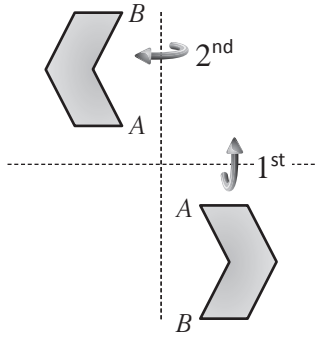


object (before) image (after)



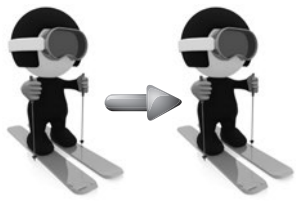
Keep equal spacing from axis.

Horizontal reflection to the right.

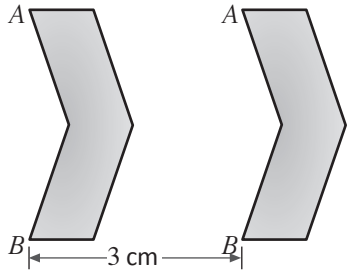


Vertical reflection up followed by a horizontal reflection left.

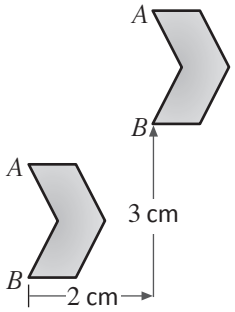
Translations (Slide) This transformation involves sliding an object either horizontally, vertically or both. Every part of the object is moved the same distance.



object (before) image (after)

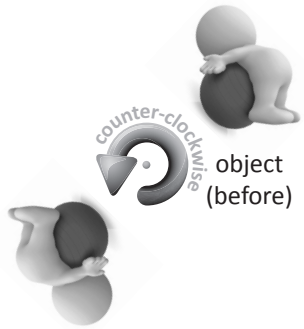


3 cm translation horizontally to the right



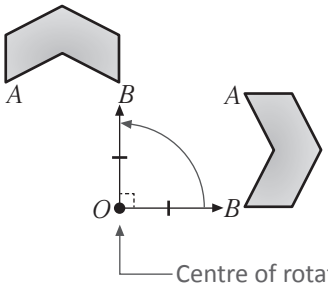
Two translations: 2 cm horizontally right, and then 3 cm vertically up

Rotations (Turn) A transformation of turning an object about a fixed point **counter-clockwise**.

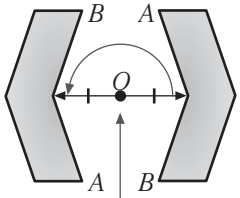


object (before)

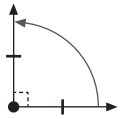
image (after)



90° rotation (or $\frac{1}{4}$ turn)

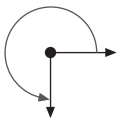


180° rotation (or $\frac{1}{2}$ turn)



90° rotation (or $\frac{1}{4}$ turn)

180° rotation (or $\frac{1}{2}$ turn)

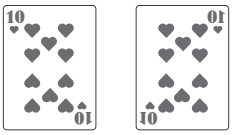


270° rotation (or $\frac{3}{4}$ turn)

Transformations

1 Identify which type of transformation each of these playing cards has undergone:

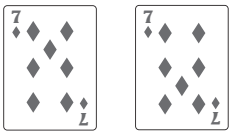
a



object image

- ☐ Reflection
- ☐ Translation
- ☐ Rotation

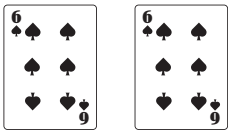
b



object image

- ☐ Reflection
- ☐ Translation
- ☐ Rotation

c

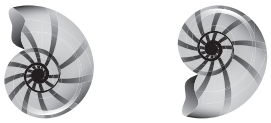


object image

- ☐ Reflection
- ☐ Translation
- ☐ Rotation

2 Each of these objects has undergone two different transformations. Tick them both.

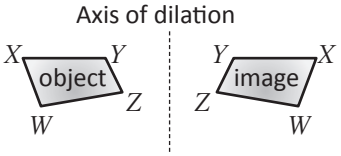
a



object image

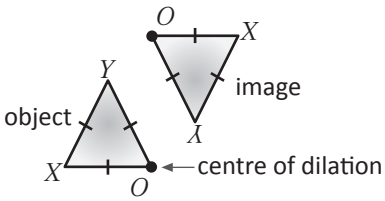
- ☐ Reflection
- ☐ Translation
- ☐ Rotation

b



- ☐ Reflection
- ☐ Translation
- ☐ Rotation

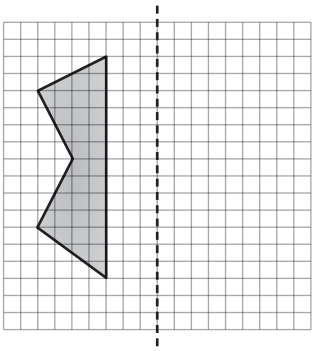
c



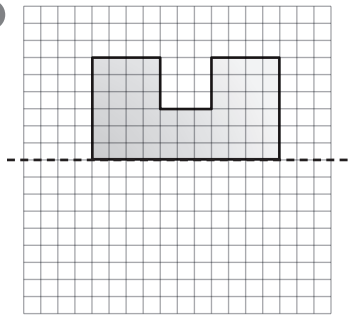
- ☐ Reflection
- ☐ Translation
- ☐ Rotation

3 Draw the image on the grids below when each of these objects are **reflected** about the given axis.

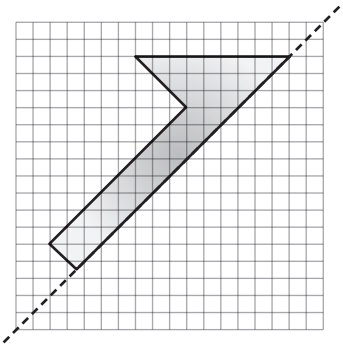
a



b

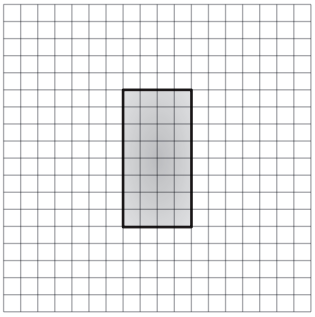


c

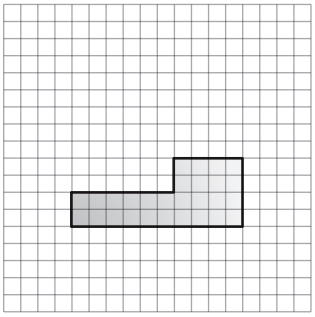


4 Draw the image on the grids below when each of these objects are **translated** by the given amounts.

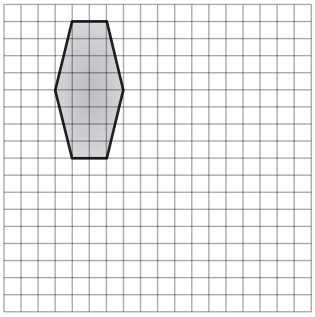
a Five squares horizontally to the left.



b Four squares vertically up.



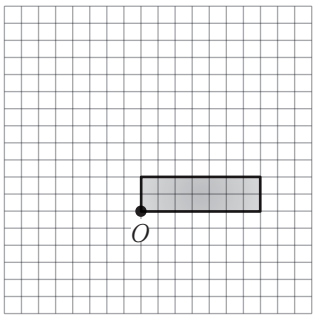
c Eight squares to the right, then six squares down.



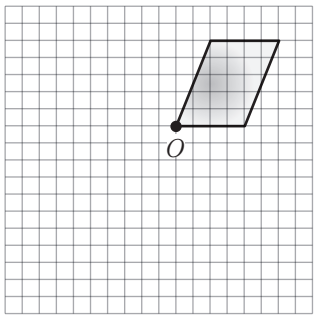
Transformations

5 Draw the image on the grids below when each of these objects are **rotated** by the given amounts.

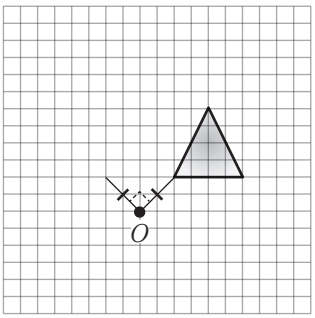
a One half turn (180° rotation).



b Three quarter turn (270° rotation).

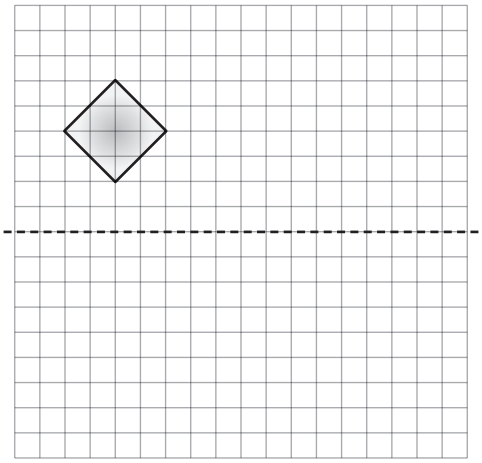


c One quarter turn (90° rotation).

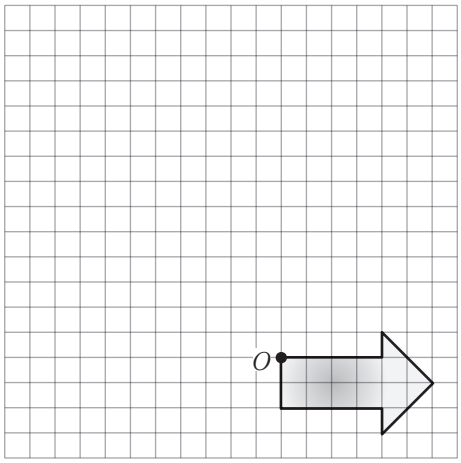


6 Draw the image on the grids below when each of these objects undergo the transformations given.

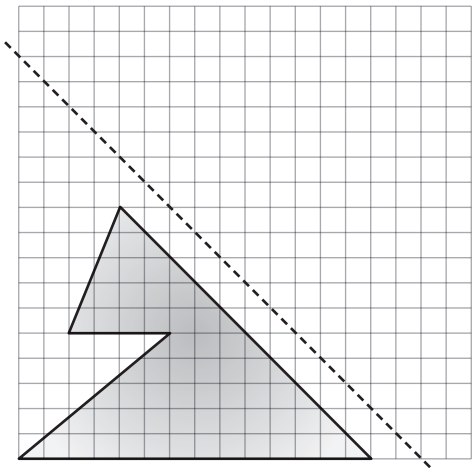
a Translate ten units to the right first then reflect down about the given axis of reflection.



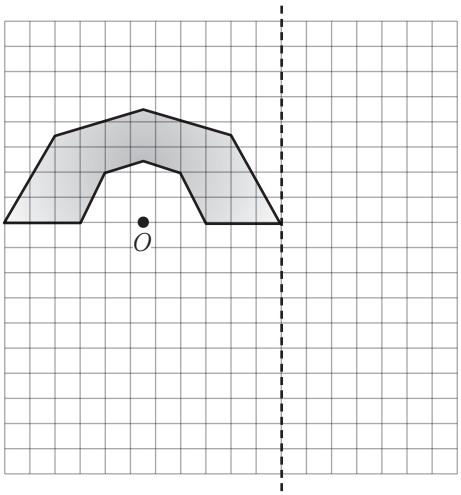
b Rotate 180° about the centre of rotation O, then translate six units up.



c Reflect about the given axis first, then tranlsate two units to the left.



d Three quarter turn (270° rotation) first, then reflect about the given axis of dilation.

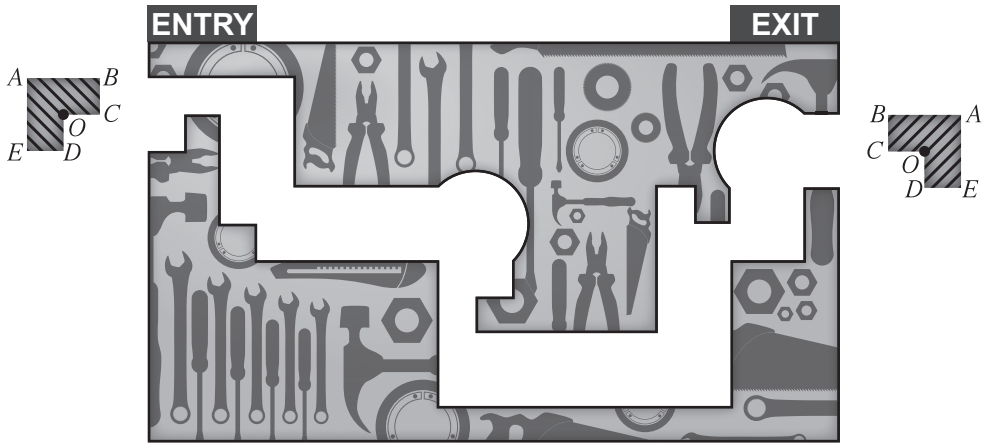


Transformations

7 Earn yourself an awesome passport stamp with this one.
The object ($ABCODE$) requires thirteen transformations to move along the white production line below. It needs to leave in the position shown at the exit for the next stage of production.

- The object must not overlap the shaded part around the production line path.
- Any of the sides AB, BC, DE and AE can be used as an axis of reflection.
- The vertex O is the only centre of rotation used at the two circle points along the path.

Describe the thirteen transformation steps used to navigate this object along the path, including the direction of transformation and the sides/points used as axes of dilation where appropriate.



(i)

(ii)

(iii)

(iv)

(v)

(vi)

(vii)

(viii)

(ix)

(x)

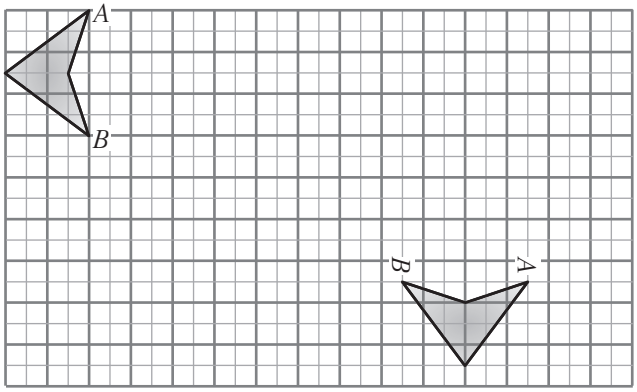
(xi)

(xii)

(xiii)

Transformations

8 For the diagram shown below, describe four different ways the final image of the object can be achieved using different transformations.



a Method 1

b Method 2

c Method 3

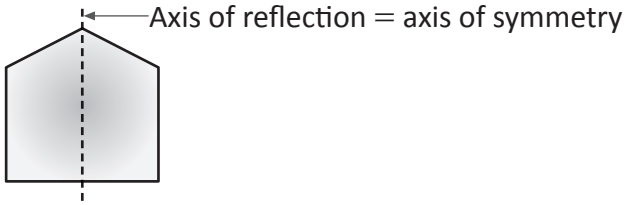
d Method 4

Reflection symmetry

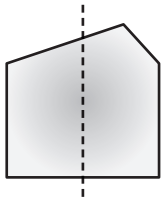
There are many types of symmetry and in this booklet we will just be focusing on three of them.

If the axis of reflection splits a shape into two identical pieces, then that shape has reflection symmetry.

The axis of reflection is then called the "axis of symmetry".

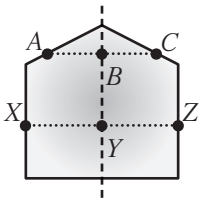


Symmetric
Shape has reflection symmetry



Asymmetric
Shape does not have reflection symmetry

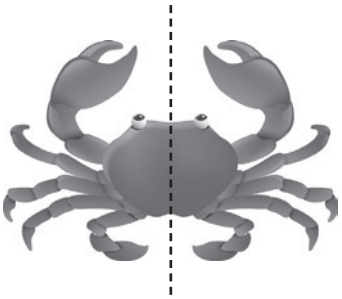
The distances from the edge of the shape to the axis of symmetry are the same on both sides of the line.



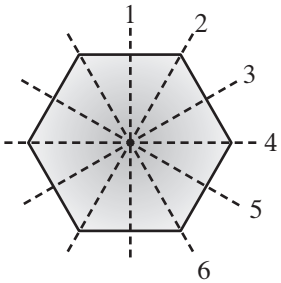
$AB = BC$ and $XY = YZ$

This shape has only one axis of symmetry. When this happens, we say the shape has bilateral symmetry.

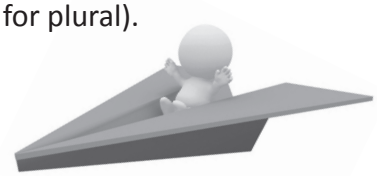
Many animals/plants or objects in nature have nearly perfect bilateral symmetry.



Other shapes can have more than one axis of symmetry (axes of symmetry for plural).



Regular Hexagon



There are 6 different ways this shape can be folded in half with both sides of the fold fitting over each other exactly. So we can say it has six-fold symmetry.

Reflection symmetry

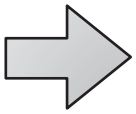
1 (i) Identify which of these shapes have reflection symmetry by ticking symmetric or asymmetric.

(ii) Draw all the axes of symmetry for those that do.

a

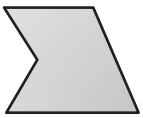
☐ Symmetric
☐ Asymmetric

b



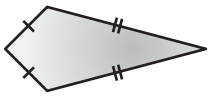
☐ Symmetric
☐ Asymmetric

c



☐ Symmetric
☐ Asymmetric

d



☐ Symmetric
☐ Asymmetric

e

☐ Symmetric
☐ Asymmetric

f

☐ Symmetric
☐ Asymmetric

g

☐ Symmetric
☐ Asymmetric

h

☐ Symmetric
☐ Asymmetric

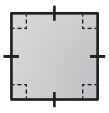
i

☐ Symmetric
☐ Asymmetric

j

☐ Symmetric
☐ Asymmetric

k



☐ Symmetric
☐ Asymmetric

l

☐ Symmetric
☐ Asymmetric

2 How many axes of reflection symmetry would these nature items have if perfectly symmetrical?

a

☐

b

☐

c

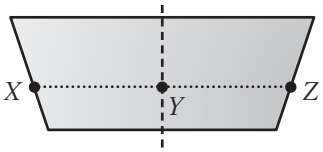
☐

d

☐

3 These shapes all have reflection symmetry. Calculate the distance between X and Y.

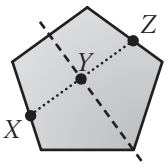
a



$YZ = 5$ cm

Distance from X to Y = ☐

b



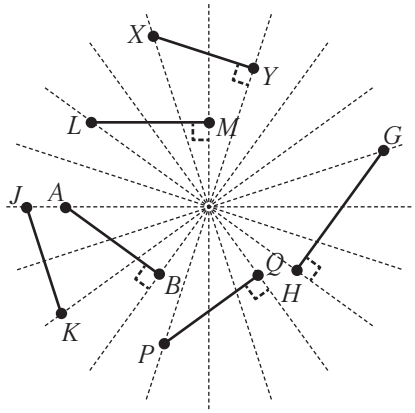
$XZ = 14$ cm

Distance from X to Y = ☐

Reflection symmetry

/20

4 Answer these questions about the symmetric web below:



a How many axes of symmetry does the web have?

b What pair of points are equidistant to LM?

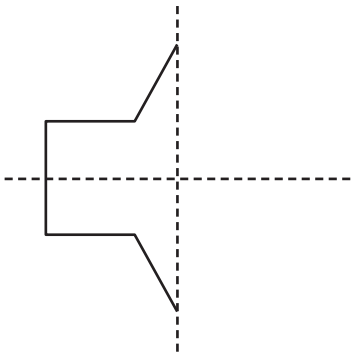
Psst: equidistant means the 'same distance'

c Briefly explain below how you decided this was the correct answer.

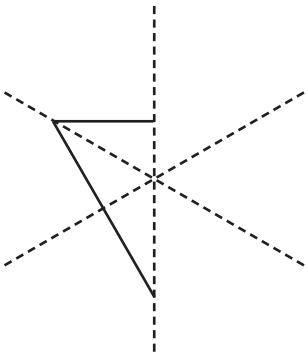
5 Complete these diagrams to produce an image with as many axes of reflective symmetry as indicated.

a Bilateral symmetry.

b Two fold symmetry.

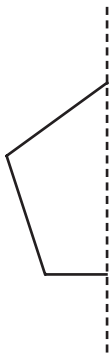


c Three axes of symmetry.

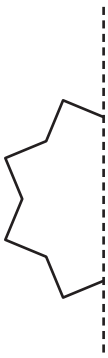


d Two axes of symmetry.

e Five-fold symmetry.
(show the other four axes)



f Eight-fold symmetry.
(show the other seven axes)

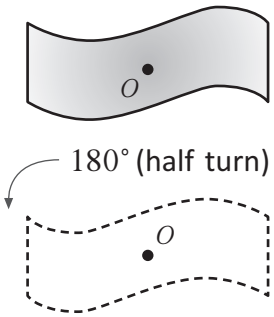


Rotational symmetry

When an object is rotated 360° (a full circle), it looks the same as it was before rotating.

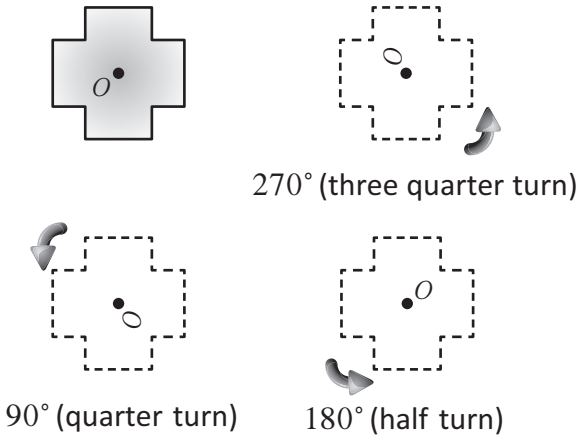
If the object looks the same again before completing a full circle, it has **rotational symmetry**.

The number of times the object 'repeats' before completing the full circle tells us the **order of rotational symmetry**.



Rotational Symmetry of order 2

i.e. it looks the same 2 times in one full rotation.



Rotational Symmetry of order 4

i.e. it looks the same 4 times in one full rotation.

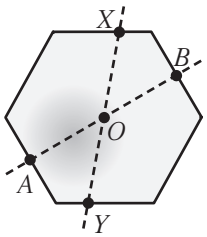
Point symmetry

This is when an object has parts the same distance away from the centre of symmetry in the opposite direction.

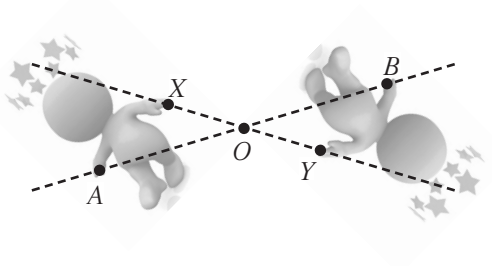
A straight line through the centre of symmetry will cross at least two points on the object.

Each pair of points crossed on opposite sides of the centre of symmetry are an equal distance away from it.

Point symmetry for one object



Point symmetry for a picture with two objects



For both diagrams: $AO = BO$ and $OX = OY$

These both have point symmetry because for every point on them, there is another point opposite the centre of symmetry (O) the same distance away.

Objects and pictures can often have both rotational and point symmetry.

Rotational and point symmetry

1 Identify which of these objects are rotationally symmetric or asymmetric.

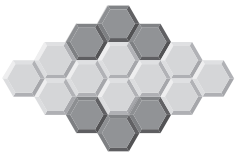
a

- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

b

- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

c



- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

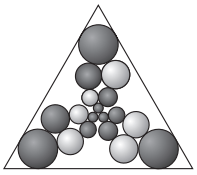
d

- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

e

- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

f



- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

2 Write the order of rotational symmetry each of these mathematical symbols have:

a ☐

b ☐

c ☐

d ☐

3 **a** All these propellers have rotational symmetry. Identify which ones also have point symmetry.

(i)

- ☐ Has point symmetry
- ☐ No point symmetry

(ii)

- ☐ Has point symmetry
- ☐ No point symmetry

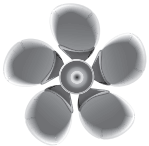
(iii)

- ☐ Has point symmetry
- ☐ No point symmetry

(iv)

- ☐ Has point symmetry
- ☐ No point symmetry

(v)



- ☐ Has point symmetry
- ☐ No point symmetry

(vi)

- ☐ Has point symmetry
- ☐ No point symmetry

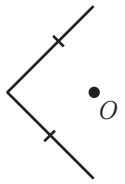
b Describe the relationship between the number of blades and the point symmetry of these propellers.

c Describe the relationship between the number of blades and the order of point symmetry for the symmetric blades.

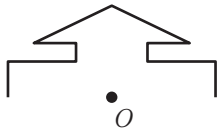
Rotational and point symmetry

4 Complete each of the half drawn shapes below to match the given symmetries.

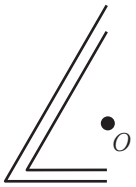
a Rotational symmetry of order 4 and also point symmetry.



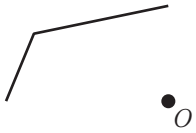
b Rotational symmetry of order 2 and also point symmetry.



c Rotational symmetry of order 3 and no point symmetry.



d Rotational symmetry of order 2 and also point symmetry.

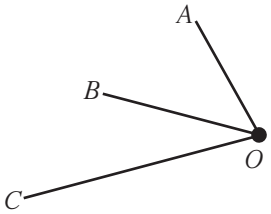


5 All the vertices shown below represent half of all the vertices of shapes which have point symmetry about the centre of rotation (O).

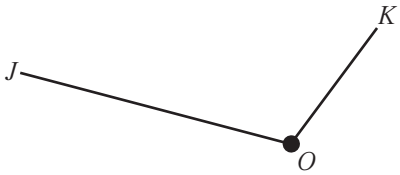
(i) Mark in the other vertices.

(ii) Draw the boundary of the whole shape.

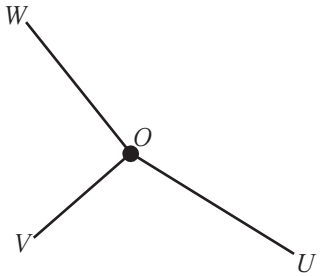
a



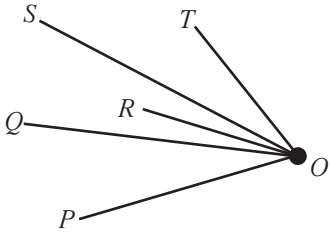
b



c



d

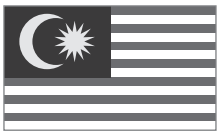


Combo time: Reflection, rotation and point symmetry

6 Identify if these flags of the world have symmetry and what type.
Include the number of folds or order of rotations for those flags with the relevant symmetry.

a Canada

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

b  Malaysia

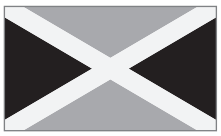
- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

c India

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

d Australia

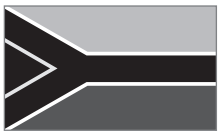
- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

e  Jamaica

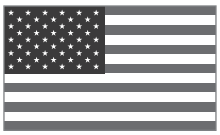
- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

f Pakistan

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

g  South Africa

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

h  United States of America

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

Combo time: Reflection, rotation and point symmetry

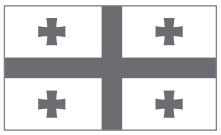
6 Identify if these flags of the world have symmetry and what type.
Include the number of folds or order of rotations for those flags with the relevant symmetry.

k Vietnam

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

l United Kingdom

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

m  Georgia

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

n New Zealand

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

o Letter 'D' signal flag

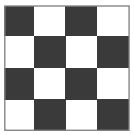
- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

p Letter 'L' signal flag

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

q Letter 'Y' signal flag

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

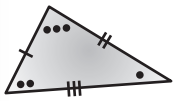
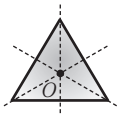
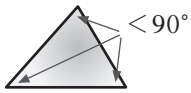
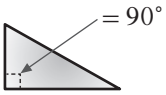
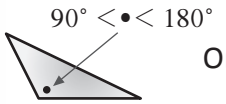
r  Letter 'N' signal flag

- ☐ Reflection symmetry with ☐ folds
- ☐ Rotational symmetry of order ☐.
- ☐ Point of symmetry.
- ☐ No symmetry

Special triangle properties

Triangles come in a number of different types, each with their own special features (properties) and names.

Here they are summarized in this table:

SHAPE	PROPERTIES
TRIANGLES	Three straight sides and internal angles.
Scalene	 All three sides have a different length. All three internal angles are a different size.
Isosceles	Two of the intenal angles have the same size. The two sides opposite the equal angles have equal lengths. 1-fold reflective symmetry. No rotational symmetry.
Equilateral	All of the internal angles have the same size of 60°. All sides have the same length.  3-fold reflective symmetry. Has rotational symmetry of order 3.
Acute angled triangle	 All of the interal angles are smaller than 90°.
Right angled triangle	 One of the internal angles is equal to 90° (i.e. one pair of sides are perpendicular to each other).
Obtuse angled triangle	 One of the internal angles is between 90° and 180°.

Determine what type of triangle is described from the information given.

(i) All internal angles are less than 90°, and it has one axis of reflection symmetry.

Isosceles triangles have one axis of reflection symmetry.

$\therefore$ It is an acute angled isosceles triangle.

(ii) All internal angels are equal and it has point symmetry.

$\therefore$ It is an equilateral triangle.

Identifying properties and naming shapes that match is called 'classifying'.

Special triangle properties

1. Classify what type of triangle is described from the information given in each of these:

   a. All internal angles are less than 90° and it has no axes of reflection.

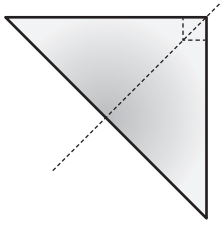
   b. One internal angle is equal to 90° and two sides are equal in length.

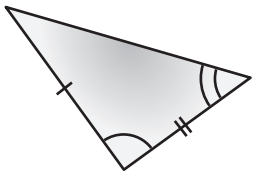
   c. One internal angle is obtuse and there is one axis of reflection.

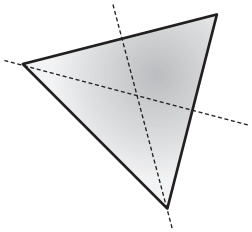
   d. Has rotational symmetry and all internal angles equal to 60°.

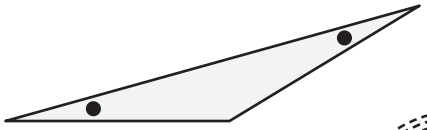
   e. No internal angles are the same size and one side is perpendicular to another.

2. Classify what type of triangle has been drawn below with only **some** properties shown.

   a.


   b.


   c.


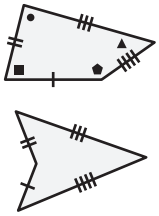
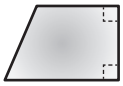
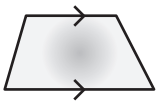
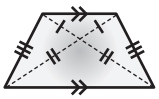
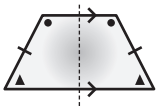
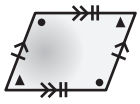
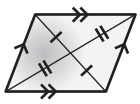
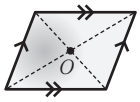
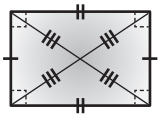
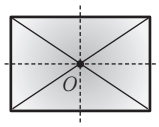
   d.


PROPERTIES SPECIAL TRIANGLE ...../20....

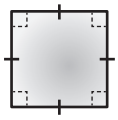
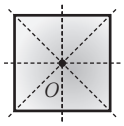
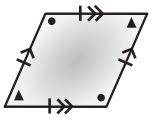
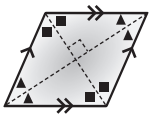
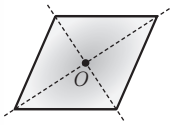
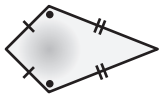
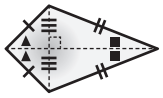
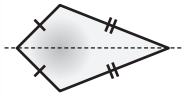
Special quadrilateral properties

Quadrilaterals exist in many different forms, each with their own special properties and names.

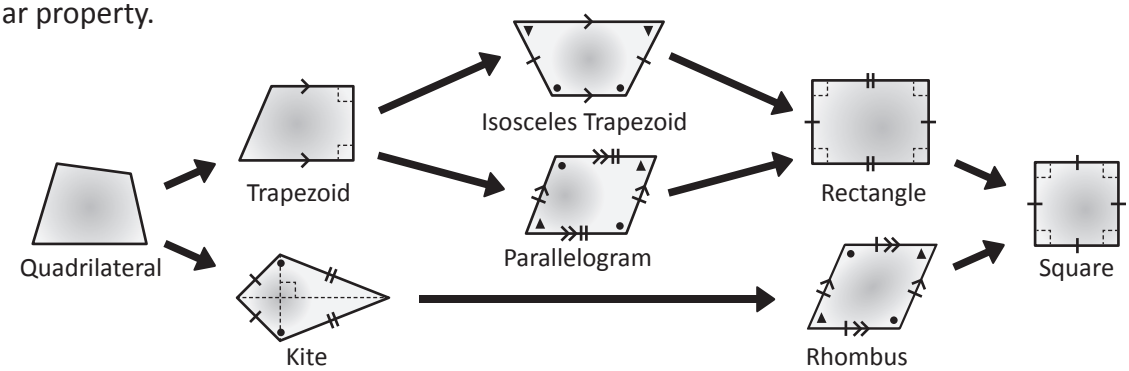
Here they are summarized in this table:

SHAPE	PROPERTIES
QUADRILATERAL	Four straight sides and internal angles.
Scalene A convex or concave quadrilateral	 All four sides have a different length. All four internal angles are a different size. No symmetry.
Trapezoid A convex quadrilateral	 Only one pair of parallel sides.  No symmetry.
Isosceles Trapezoid	 Non-parallel sides are the same length. Diagonals cut each other into equal ratios.  Two pairs of equal internal angles with common arms. 1 axis of reflective symmetry.
Parallelogram A convex Qaudrilateral	 Opposite sides are parallel. Opposite sides are equal in length. Diagonally opposite internal angles are equal.  Diagonals bisect each other (cut each other exactly in half).  No axis of reflective symmetry. Rotational symmetry of order 2 and point symmetry at the intersection of the diagonals O.
Rectangle A convex, equiangular quadrilateral	Opposite sides are parallel. Opposite sides are equal in length. All internal angles $= 90^\circ$.  Diagonals are equal in length. Diagonals bisect each other (cut each other exactly in half).  2-fold reflective symmetry. Rotational symmetry of order 2 and point symmetry at the intersection of the diagonals O.

Special quadrilateral properties

SHAPE	PROPERTIES
Square **A regular quadrilateral**	 Opposite sides are parallel. Opposite sides are the same length. All internal angles $= 90^\circ$. Diagonals bisect each other. Diagonals bisect each internal angle. Diagonals cross at right angles to each other (perpendicular).  4-fold reflective symmetry. Rotational symmetry of order 4 and point symmetry at the intersection of the diagonals O.
Rhombus A convex quadrilateral	 Opposite sides are parallel. All sides are the same length. Diagonally opposite internal angles are the same.  Diagonals bisect each other. Diagonals bisect each internal angle. Diagonals cross at right angles to each other (perpendicular).  2-fold reflective symmetry. Rotational symmetry of order 2 and point symmetry at the intersection of the diagonals O.
Kite A convex quadrilateral	 Two pairs of adjacent, equal sides. Internal angles formed by unequal sides are equal.  Shorter diagonal is bisected by the longer one. Longer diagonal bisects the angles it passes through. Diagonals are perpendicular to each other.  1-fold reflective symmetry. No Rotational symmetry.

This diagram shows how each quadrilateral relates to the previous one which shares one similar property.



Special quadrilateral properties

1 Classify what special quadrilateral is being described from the information given in each of these:

a Two pairs of equal sides, all internal angles are right-angles and has 2-fold reflective symmetry.

b One pair of parallel sides and one pair of opposite equal sides.

c Two pairs of equal internal angles with the diagonals the only axes of reflective symmetry.

d One pair of parallel sides and one pair of opposite equal sides.

e Diagonals bisect each other and split all the internal angles into pairs of 45°.

f Perpendicular diagonals and no rotational symmetry.

2 Write down two differences between each of these special quadrilaterals:

a A square and a rectangle.

b A rectangle and a parallelogram.

c A parallelogram and a rhombus.

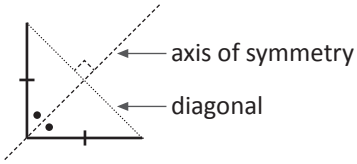
d A rhombus and a kite.

e A rhombus and a square.

f A kite and an isosceles trapezoid.

PROPERTIES * SPECIAL QUADRILATERAL ...../...../20....

3 A quadrilateral has been partially drawn below. Draw and name the three possible quadrilaterals this diagram could have been the start of according to the given information.



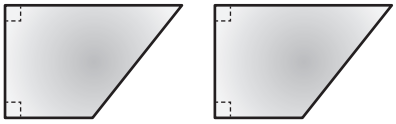
a

b

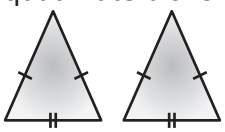
c

Combo time! Special quadrilateral and triangles

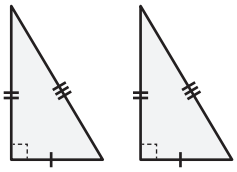
1. These two identical trapezoids can be transformed and combined to make two special quadrilaterals. Explain the transformation used, and then name and draw the new quadrilateral formed.



2. These two equal isosceles triangles can be transformed and combined to make two special quadrilaterals. Explain the transformation used, then name and draw the two special quadrilaterals formed.



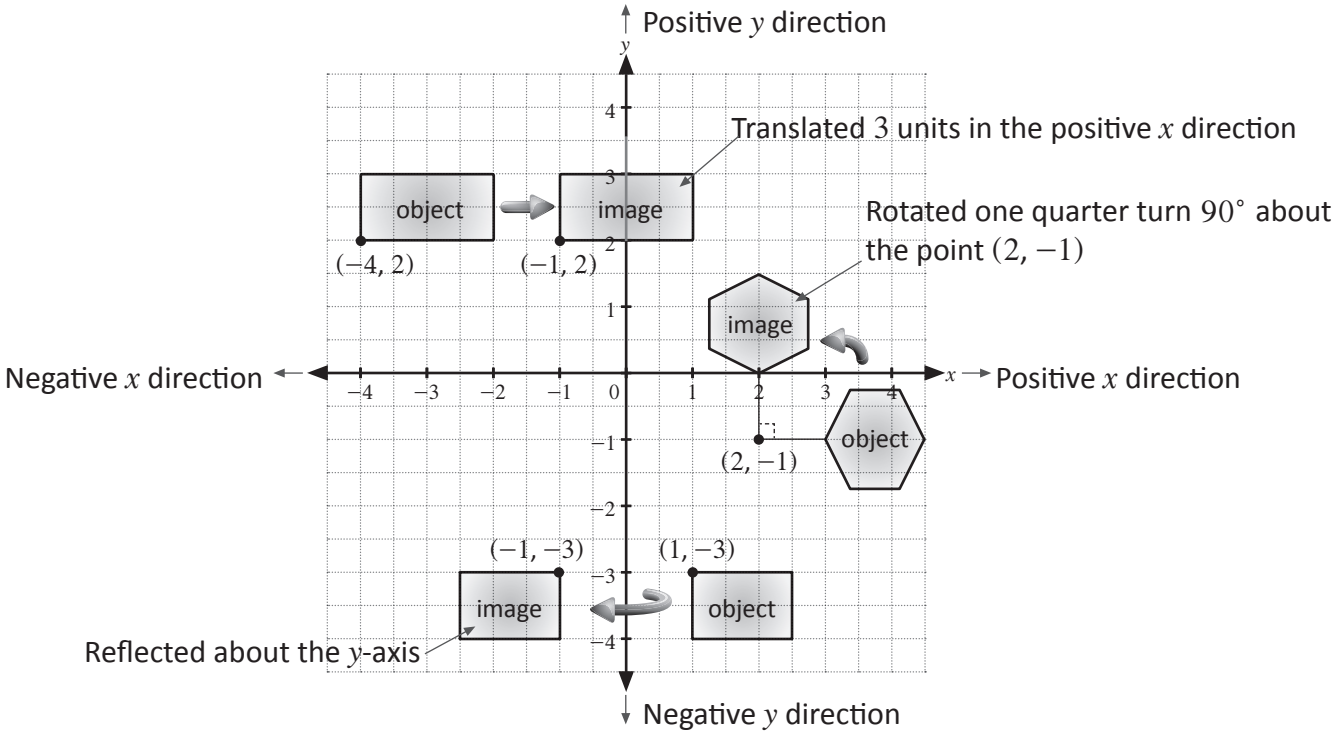
3. Draw all the different quadrilaterals that can be formed using these two identical right-angled scalene triangles.



Transformations on the Cartesian number plane

Just as grids were used earlier to help transform shapes, the number plane can also be used.

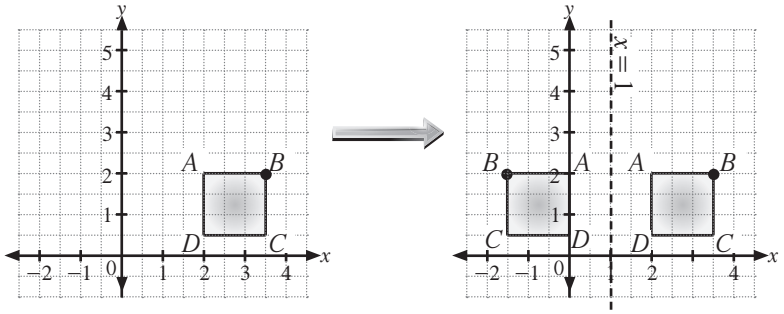
The coordinates of vertices help us locate and move objects accurately.



Same methods apply as before, this time including the new coordinates of important points.

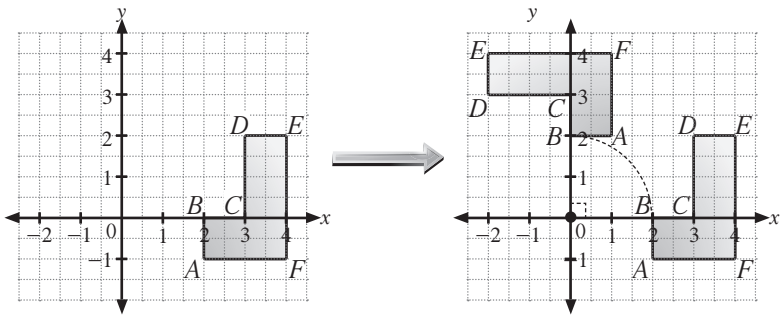
Determine the new coordinates for the points after these translations

(i) The coordinates of 'B' after $ABCD$ is reflected about the line $x = 1$.



New coordinates for B are $(-1.5, 2)$

(ii) The coordinates of 'E' after the shape $ABCDEF$ is rotated 90° about the origin $(0, 0)$.

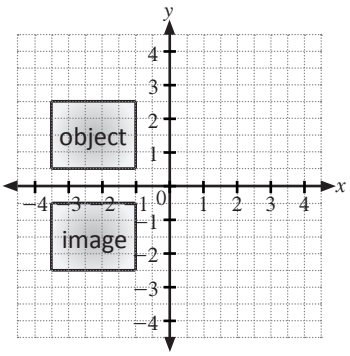


New coordinates for E are $(-2, 4)$

Transformations on the Cartesian number plane

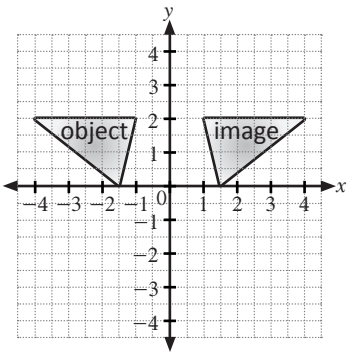
1 All these images are reflections of the object.
Choose whether the reflection was vertical (up/down), horizontal (right/left) or both (diagonally).

a



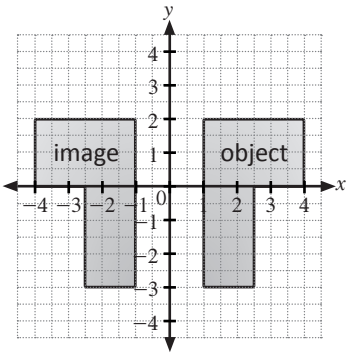
Reflected
- ☐ Vertically
- ☐ Horizontally
- ☐ Diagonally

b



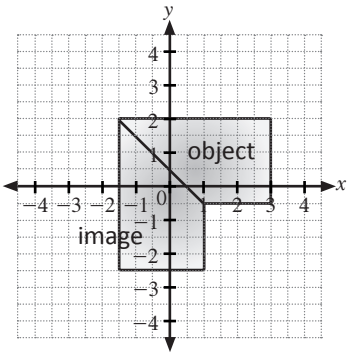
Reflected
- ☐ Vertically
- ☐ Horizontally
- ☐ Diagonally

c



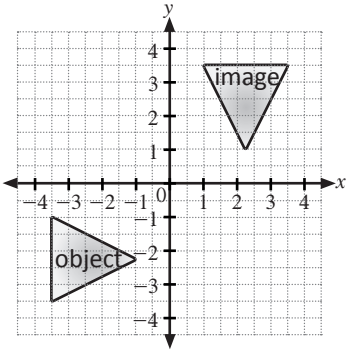
Reflected
- ☐ Vertically
- ☐ Horizontally
- ☐ Diagonally

d



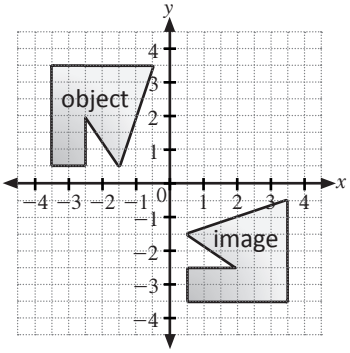
Reflected
- ☐ Vertically
- ☐ Horizontally
- ☐ Diagonally

e



Reflected
- ☐ Vertically
- ☐ Horizontally
- ☐ Diagonally

f

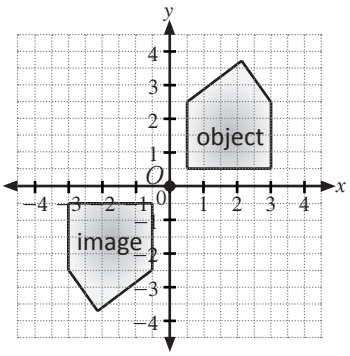


Reflected
- ☐ Vertically
- ☐ Horizontally
- ☐ Diagonally

Transformations on the Cartesian number plane

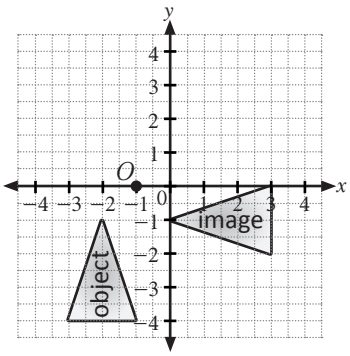
2 All these images are rotations of the object.
Choose whether the rotation is 90°, 180° or 270° about the given point of rotation labelled O.

a



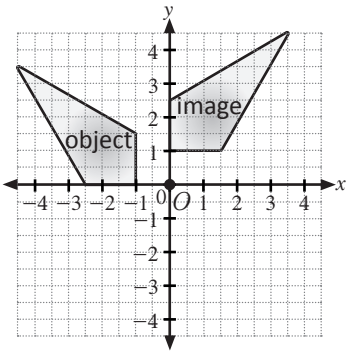
☐ 90° ☐ 180° ☐ 270° rotation

b



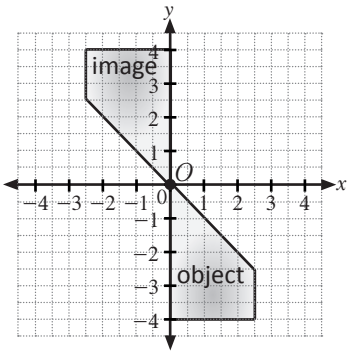
☐ 90° ☐ 180° ☐ 270° rotation

c



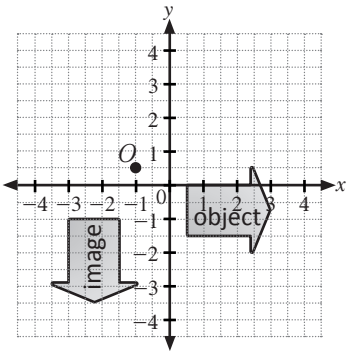
☐ 90° ☐ 180° ☐ 270° rotation

d



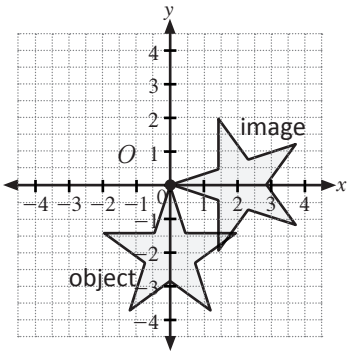
☐ 90° ☐ 180° ☐ 270° rotation

e



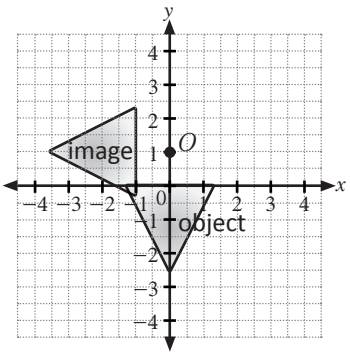
☐ 90° ☐ 180° ☐ 270° rotation

f



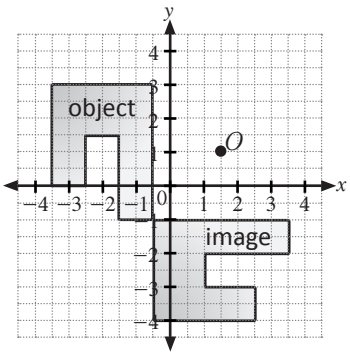
☐ 90° ☐ 180° ☐ 270° rotation

g



☐ 90° ☐ 180° ☐ 270° rotation

h

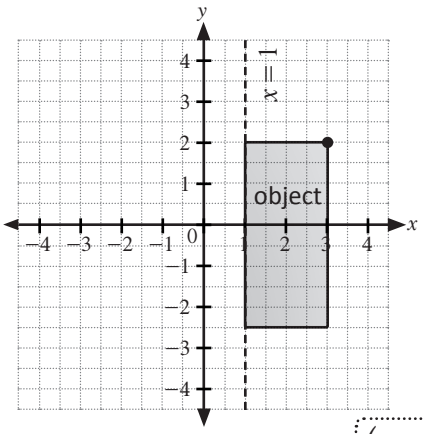


☐ 90° ☐ 180° ☐ 270° rotation

Transformations on the Cartesian number plane

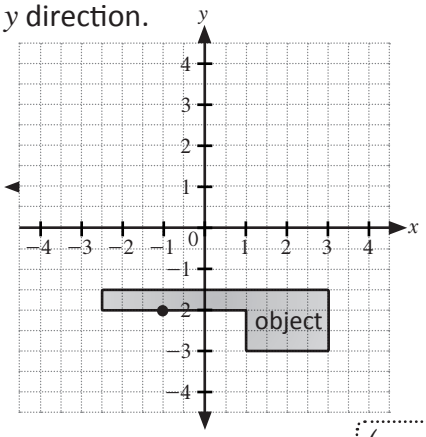
3 (i) Draw the image for the requested transformations on the number planes below.
(ii) Write down the new coordinates for the dot marked on each object.

a Reflect object about the line $x = 1$.



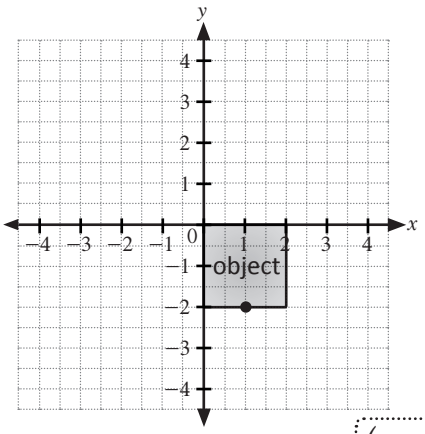
New coordinates for dot = (,)

b Translate the object four units in the positive y direction.



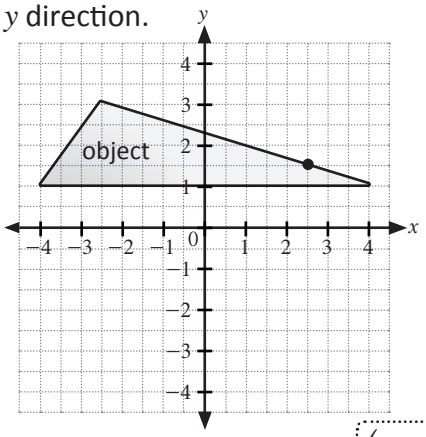
New coordinates for dot = (,)

c Rotate the object $180°$ about the $(0, 0)$.



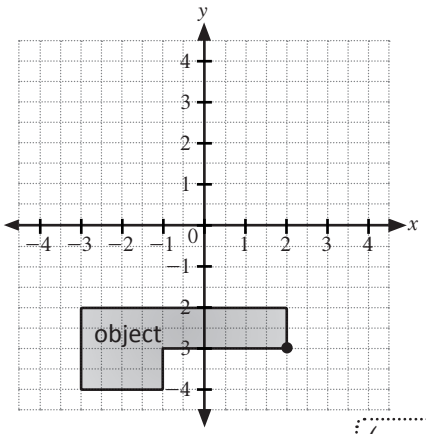
New coordinates for dot = (,)

d Translate the object four units in the negative y direction.



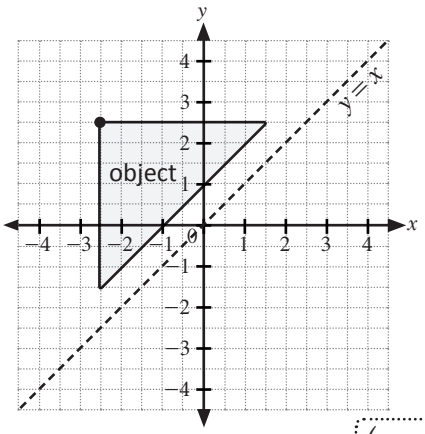
New coordinates for dot = (,)

e Reflect object about the x-axis.



New coordinates for dot = (,)

f reflect object about the given axis line, $y = x$.

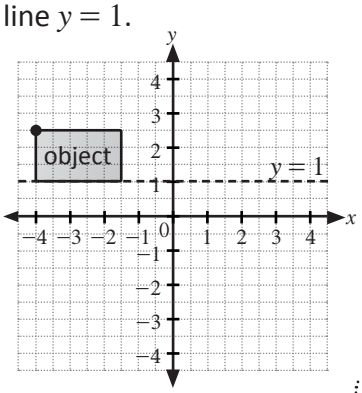


New coordinates for dot = (,)

Transformations on the Cartesian number plane

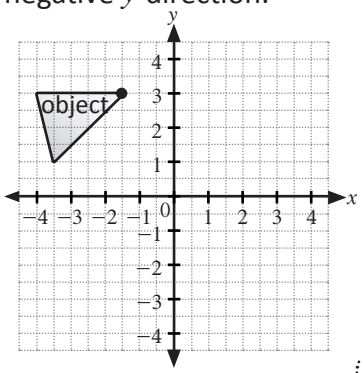
4 (i) Draw the image for the requested double transformations on the number planes below.
(ii) Write down the new coordinates for the dot marked on each image.

a Translate object 3 units in the positive x-direction and then reflect about the line $y = 1$.



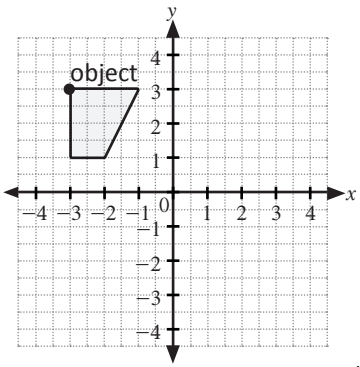
New coordinates for dot = (,)

b Rotate the object one quarter turn about the point $(-1,\ 3)$ then translate 2.5 units in the negative y-direction.



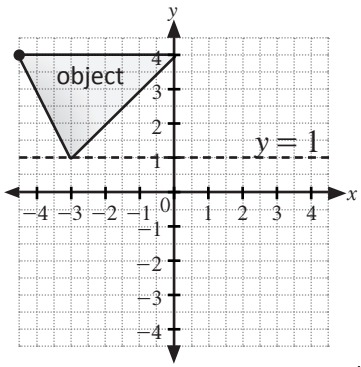
New coordinates for dot = (,)

c Rotate object $270°$ about the point $(-1,\ 1)$ and then reflect about the x-axis.



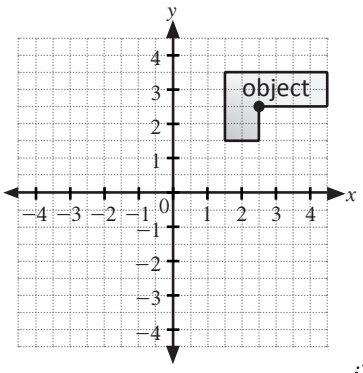
New coordinates for dot = (,)

d Reflect the object about the y-axis, and then reflect about the line $y = 1$.



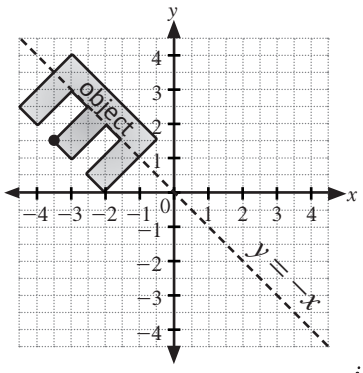
New coordinates for dot = (,)

e Reflect object about the y-axis then rotate $180°$ about the origin $(0,\ 0)$.



New coordinates for dot = (,)

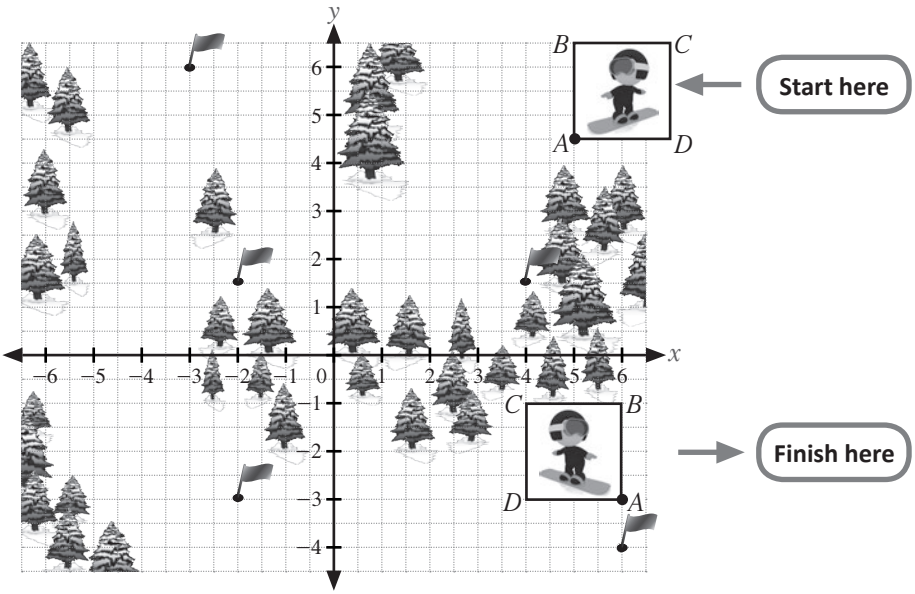
f Translate the object 2.5 units in the negative y-direction and then reflect about the line $y = -x$.



New coordinates for dot = (,)

Transformations on the Cartesian number plane

5 A player in a snow sports game can only use transformations to perform tricks and change direction to get through the course marked by trees.
Points are deducted if trees are hit. Points are awarded when the corner dot marked 'A' passes directly over coordinates marked with flags on the course.
The dimensions of the player are a square with sides two units long.
Write down the steps (including the coordinates of point A after each transformation) a player can take to get maximum points from start to finish.

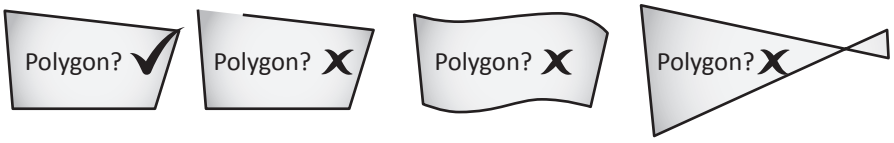


Here is what you need to remember from this topic on polygons

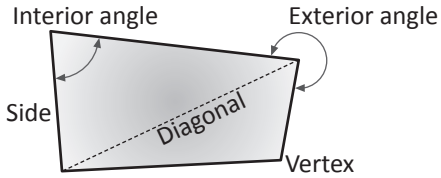
Polygons

Polygons are just any closed shape with straight lines which don't cross.
Like a square or triangle.

All polygons need at least three sides to form a closed path.



Shapes which are/are not polygons



Parts of a polygon

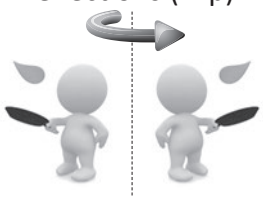
Types of polygons:

Convex	Equilateral	Regular	Cyclic
All interior angles are $< 180°$	All sides are the same length	All interior angles are equal All sides are the same length They are cyclic polygons	All vertices/corner points lie on the edge (circumference) of the same circle.
Concave	**Equiangular**		
Has an interior angle $> 180°$	All interior angles are equal		

Sides	Polygon name	Sides	Polygon name	Sides	Polygon name
3	Trigon (triangle)	4	Tetragon	5	Pentagon
6	Hexagon	7	Heptagon	8	Octagon
9	Nonagon	10	Decagon	11	Hendecagon
12	Dodecagon	15	Pentadecagon	20	Icosagon

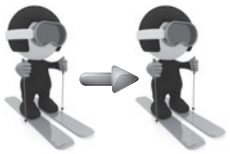
Transformations

Reflections (Flip)



object image

Translations (Slide)



object image

Rotations (Turn)

image

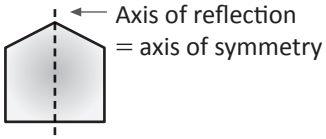
$90°$ rotation (or $\frac{1}{4}$ turn)

$180°$ rotation (or $\frac{1}{2}$ turn)

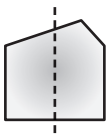
$270°$ rotation (or $\frac{3}{4}$ turn)

Reflection Symmetry

Where an axis of reflection splits an object into two identical pieces.

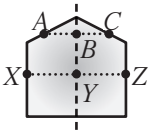


Symmetric: Shape has reflection symmetry



Asymmetric: Shape does not have reflection symmetry

The distances from the edge of the shape to the axis of symmetry are the same on both sides of the line.



$AB = BC$ and $XY = YZ$

Rotational Symmetry

If an object looks the same during a rotation before completing a full circle, it has **rotational symmetry**. The number of times the object 'repeats' before completing the full circle tells us the **order of rotational symmetry**.

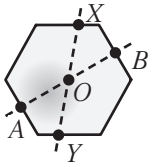
90° (quarter turn) 180° (half turn) 270° (three quarter turn)

Rotational Symmetry of order 4 as it looks the same four times within one full rotation.

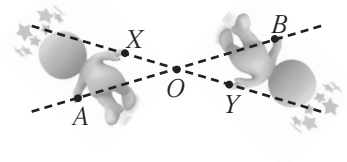
Point Symmetry

These objects have point symmetry because for every point on them, there is another point opposite the centre of symmetry (O) the same distance away.

Point symmetry for one object



Point symmetry for two object



For both diagrams:
$AO = BO$ and $OX = OY$

Special Triangles and Quadrilaterals (summary of key sides and angle differences only)

Triangles

Scalene	Isosceles	Equilateral	Right angled triangle
No equal sides or angles	1 pair of equal sides & angles	All sides and angles equal	1 internal angle $= 90^\circ$

Acute	Obtuse
All internal angles $< 90^\circ$	One internal angle between 90° and 180°

Quadrilaterals

Scalene	Trapezoid	Isosceles Trapezoid
No equal sides or angles.	At least 1 pair of parallel sides.	At least 1 pair of parallel sides. Non-parallel sides equal in length.

Parallelogram	Rectangle	Square
Opposite sides equal in length and parallel to each other.	Opposite sides equal in length and parallel to each other. All internal angles $= 90^\circ$.	All sides equal in length and opposite sides parallel to each other. All internal angles $= 90^\circ$.

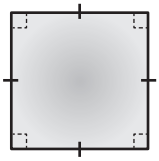
Rhombus	Kite
All sides equal in length and opposite sides parallel to each other. Diagonally opposite internal angles equal.	Two pairs of adjacent equal sides. Angles opposite short diagonal equal.

For a more detailed summary, see pages 19, 21 and 22 of the booklet.

Polygons

1. a Polygon b Not a polygon
c Not a polygon d Polygon
e Not a polygon f Not a polygon
g Not a polygon h Not a polygon

2. a Convex pentagon
b Convex, cyclic trigon (or triangle)
c Convex, cyclic, equiangular hexagon
d Concave quadrilateral
e Convex, equilateral, equiangular, cyclic, regular tetragon
f Concave, equilateral hexadecagon

3. a

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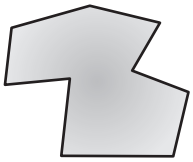

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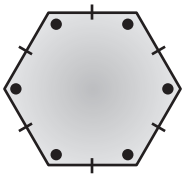
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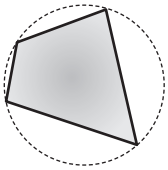
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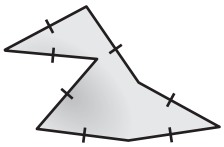


5. A cyclic quadrilateral cannot have any concave angles.

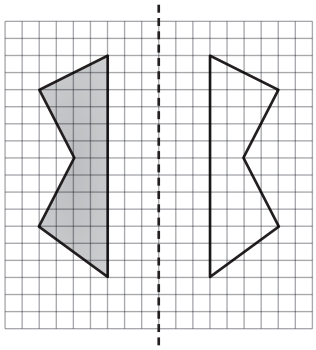
6. a A tetragon with one pair of equal sides next to each other (adjacent). The angle diagonally opposite the angle formed by the two equal sides is a right angle (or equal to $90°$).

b A peanut-shaped equilateral concave octagon with two internal reflex angles.

Transformations

1. a Translation Rotation b Rotation c Translation

2. a Reflection Rotation b Reflection Translation c Translation Rotation

3. a

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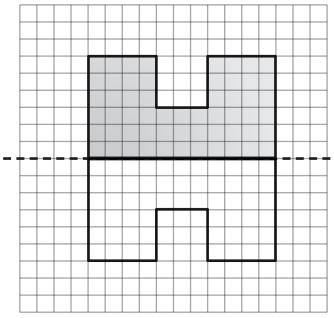
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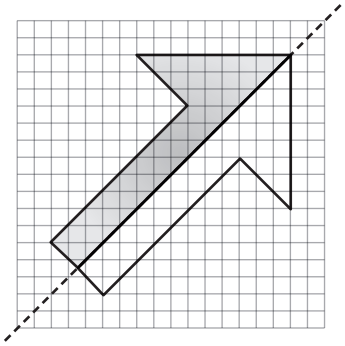

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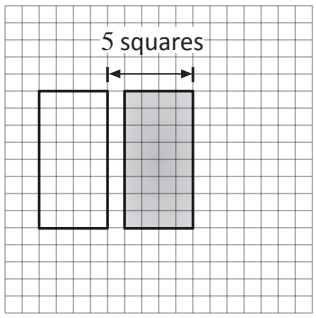
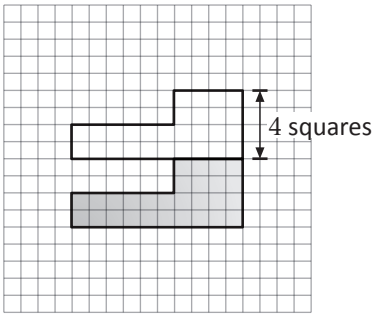


Transformations

4. **b**



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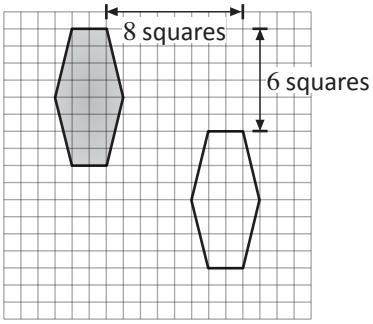
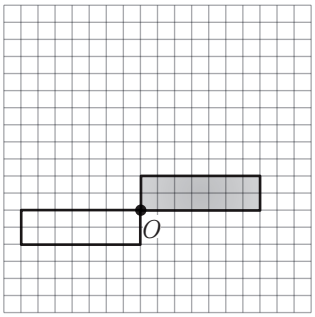
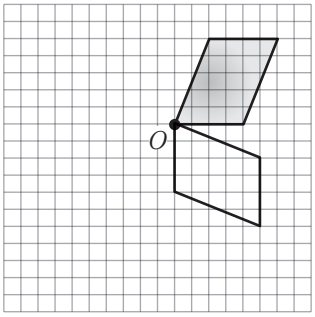


5. **a**



b



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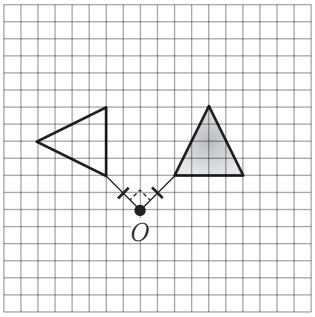
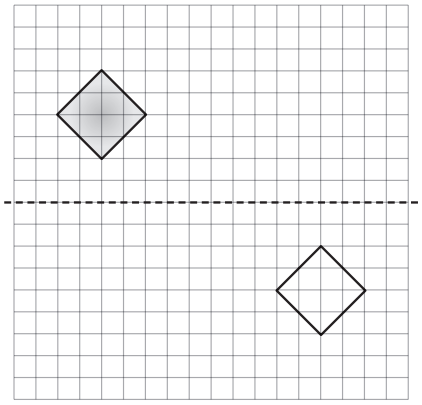


Transformations

6. **a**



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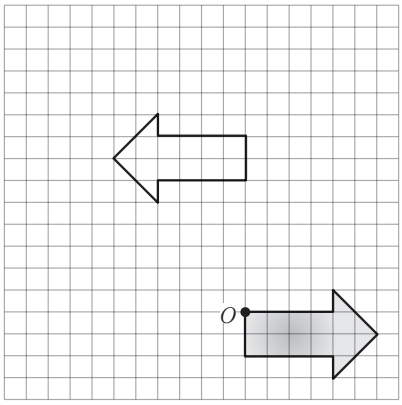

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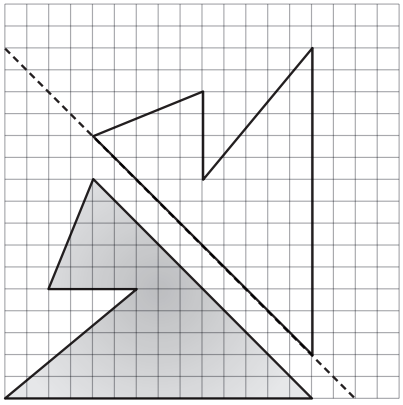

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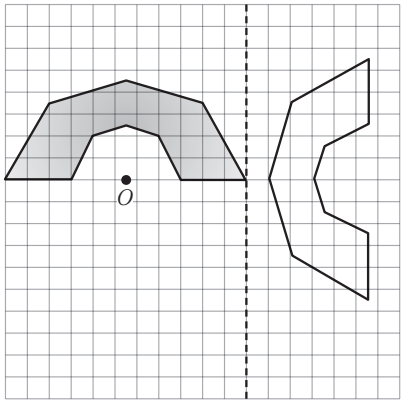


Transformations

7. (i) Translate shape horizontally to the right.

(ii) Reflect shape horizontally to the right using the edge BC as the axis of reflection.

(iii) Translate shape vertically down.

(iv) Translate shape horizontally to the right.

(v) Rotate shape one quarter turn ($90°$) about the vertex O.

(vi) Translate shape vertically down.

(vii) Reflect shape vertically down using the edge BC as the axis of reflection.

(viii) Translate shape horizontally to the right.

(ix) Translate shape vertically up.

(x) Reflect shape horizontally to the right using the edge DE as the axis of reflection.

(xi) Translate shape vertically up.

(xii) Rotate shape one quarter turn ($90°$) about the vertex O.

(xiii) Translate shape horizontally to the right.

8. a Method 1
- Translate shape horizontally 7.5 units to the right.
- Reflect the shape about the axis horizontally to the right.
- Rotate shape $\frac{3}{4}$ of a turn ($270°$) about the point .
- Translate shape vertically down 2.5 units.

b Method 2
- Translate shape horizontally 7.5 units to the right.
- Rotate shape $\frac{1}{4}$ of a turn ($90°$) about the point B.
- Translate shape vertically down 2.5 units.
- Reflect the shape horizontally to the right about a vertical axis passing through B.

Transformations

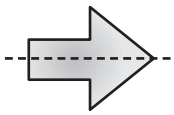
8. c Method 3
- Reflect the shape vertically down about a horizontal axis passing through B.
- Translate shape vertically down 2.5 units.
- Rotate shape $\frac{1}{4}$ of a turn ($90°$) about the point B.
- Translate shape horizontally 7.5 units to the right

d Method 4
- Rotate shape $\frac{1}{4}$ of a turn ($90°$) about the point A.
- Translate shape horizontally 4.5 units to the right.
- Reflect the shape horizontally to the right about a vertical axis passing through B.
- Translate shape vertically 5.5 units down.

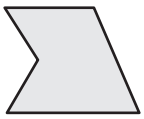
Reflection symmetry

1. a

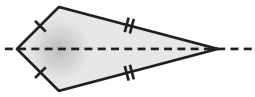
Symmetric

b 

Symmetric

c 

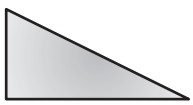
Asymmetric

d 

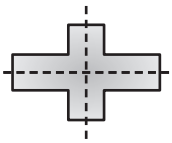
Symmetric

e

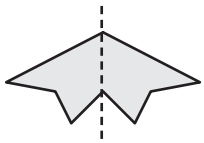
Symmetric

f 

Asymmetric

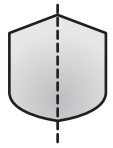
g 

Symmetric

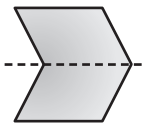
h 

Symmetric

Reflection symmetry

1. i 

Symmetric

j 

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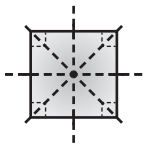
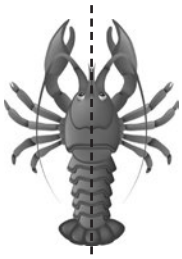
k 

Symmetric

l

Symmetric

2. a 

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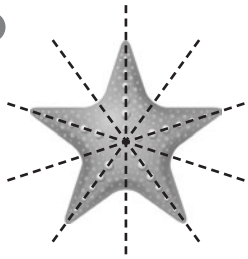
b 

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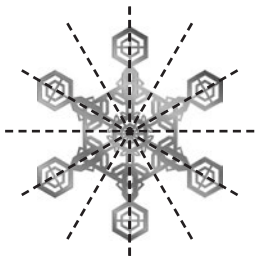
c 

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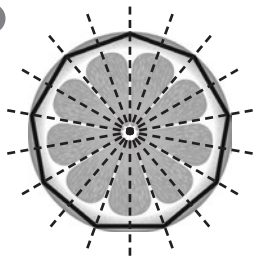
d 

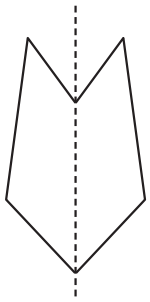
9

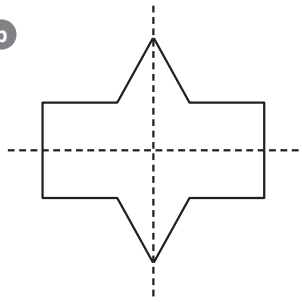
3. a 5 cm b 7 cm

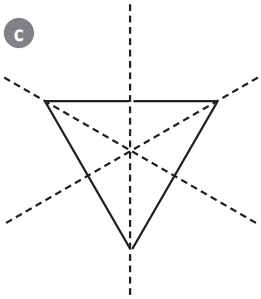
4. a 10 b P and Q

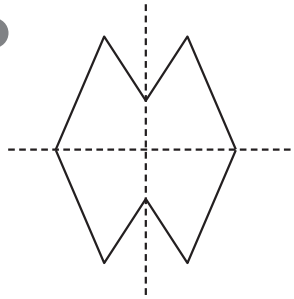
c Shapes with reflective symmetry have points equidistant on the other side of the axis. Points P and Q are just like LM, they start on an axis between the 3rd and 4th curved circle, move perpendicular to the axis and end on a point of the fifth circle three axes across.

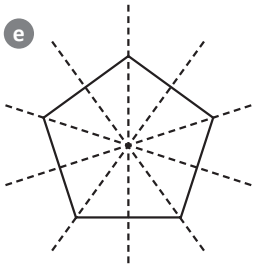
Reflection symmetry

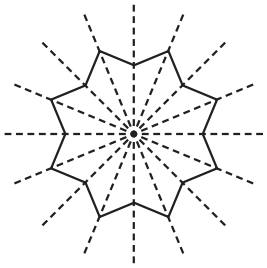
5. a 

b 

c 

d 

e 

f 

Rotational and point symmetry

1. a Rotationally symmetric

b Rotationally asymmetric

c Rotationally symmetric

d Rotationally symmetric

e Rotationally symmetric

f Rotationally asymmetric

2. a 2 b 4 c 2 d 4

3. a (i) No point symmetry (ii) Has point symmetry

(iii) Has point symmetry (iv) Has point symmetry

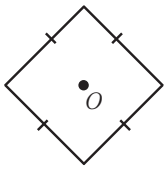
(vi) No point symmetry (vii) No point symmetry

Rotational and point symmetry

3. **b** If there is an even number of blades, the shape has point symmetry, if there is an odd number of blades, it doesn't.

c The number of blades on the even blade propellers is equal to the order of rotational symmetry.

4. **a**



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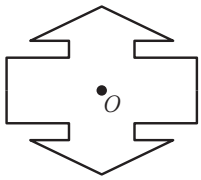
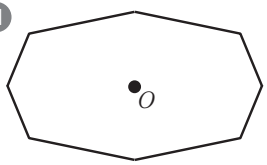
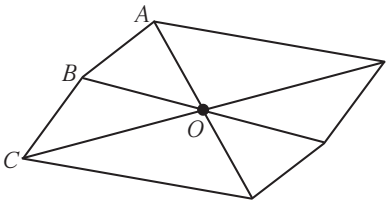


c

d



5. **a**



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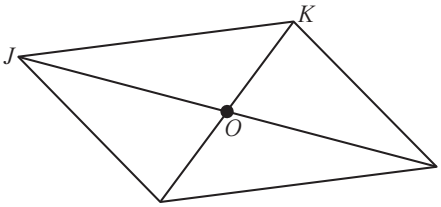

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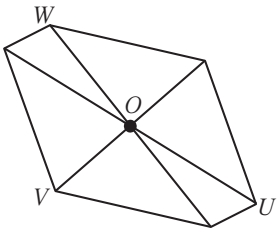
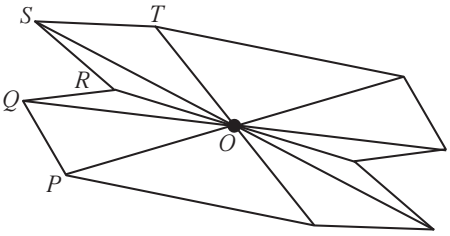


d



Combo time: Reflection, rotation and point symmetry

6. **a** Reflection symmetry with 1 folds

b No symmetry

c Reflection symmetry with 1 folds

d No symmetry

e Reflection symmetry with 2 folds
Rotational symmetry of order 2.
Point of symmetry.

f No symmetry

g No symmetry

h No symmetry

i Reflection symmetry with 1 folds

j Rotational symmetry of order 2.

k Reflection symmetry with 2 folds
Rotational symmetry of order 2.
Point of symmetry.

l No symmetry

m Reflection symmetry with 2 folds
Rotational symmetry of order 2.
Point of symmetry.

n Reflection symmetry with 2 folds
Rotational symmetry of order 2.
Point of symmetry.

o No symmetry

p Reflection symmetry with 2 folds
Rotational symmetry of order 2.
Point of symmetry.

Special triangle properties

1. **a** Acute-angled scalene triangle.

b Right-angled isosceles triangle.

c Obtuse-angled isosceles triangle.

d Equilateral triangle.

e Right-angled scalene triangle.

Special triangle properties

2. a Right-angled isosceles triangle

b Scalene triangle c Equilateral triangle

d Isosceles triangle

Special quadrilateral properties

1. a Rectangle b Isosceles trapezium

c Rhombus d Kite

e Square f Kite

2. a
- Squares have all sides equal, not just opposite sides.
- Squares have 2 more folds of reflective symmetry.

b
- Rectangles have all internal angles equal.
- Rectangles have reflective symmetry.

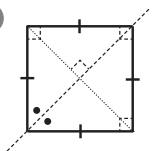
c
- Rhombus' have all sides equal, not just opposite sides.
- Diagonals of a rhombus are perpendicular.
- Diagonals of a rhombus bisect the angles.

d
- Opposite sides parallel in a rhombus.
- All sides equal in a rhombus.
- Both diagonals bisect the angles in a rhombus.
- Kite has only one pair of equal opposite angles.

e
- Rhombus has all sides equal, not just opposite sides
- Diagonals of a rhombus are perpendicular.
- Squares have 4 folds of symmetry, rhombus only 2.

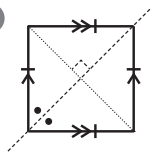
f
- Opposite sides parallel in a rhombus.
- Both diagonals bisect the angles in a rhombus
- Opposite angles equal in a rhombus, adjacent ones equal in isosceles trapezium.

3. a



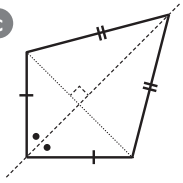
Square

b



Rhombus

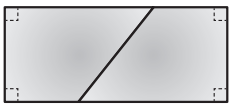
c



Kite

Combo time: Special quadrilateral and triangles

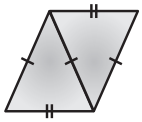
1.



Rotate one shape a half-turn and translate until the non-perpendicular sides are common.Or reflect one horizontally then transpose.

Rotate one shape a half-turn and translate until the perpendicular sides are common. Or reflect one horizontally and then vertically, then translate.

2.



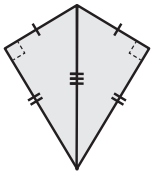
Parallelogram

Reflect one vertically up/down and then transpose until one pair of equal sides are shared.

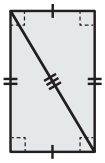
Rhombus

Reflect one vertically down and then transpose until the shorter sides are shared.

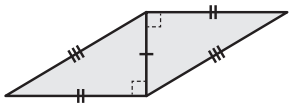
3.



Kite



Rectangle



Parallelogram

Transformation on the Cartesian number plane

1. a Reflected Vertically ↓

b Reflected Horizontally →

c Reflected Horizontally ←

d Reflected Diagonally ↙

e Reflected Diagonally ↗

f Reflected Diagonally ↘

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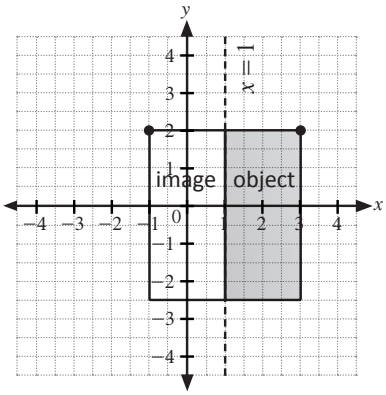
Transformation on the Cartesian number plane

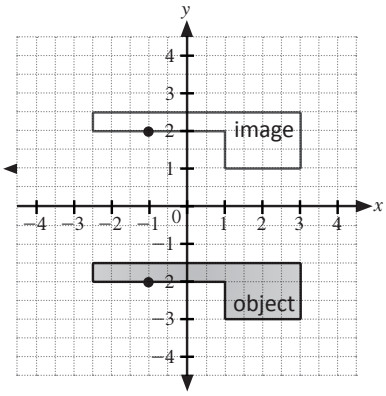
2. a 180° rotation b 90° rotation
c 270° rotation d 180° rotation
e 270° rotation f 90° rotation
g 270° rotation h 90° rotation

3. a



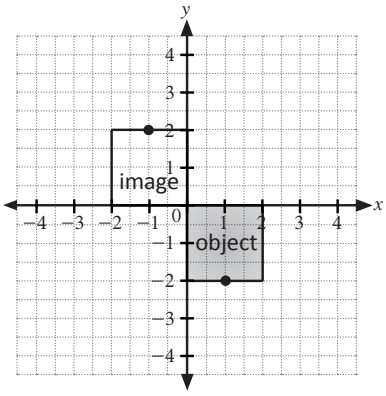
New coordinates for dot = $(-1, 2)$

b



New coordinates for dot = $(-1, 2)$

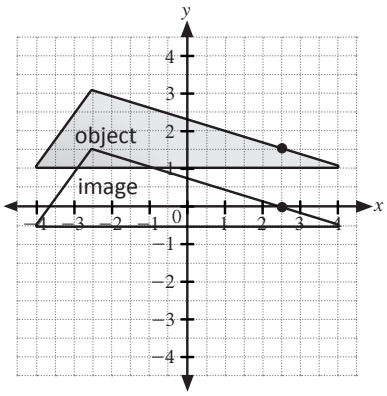
c



New coordinates for dot = $(-1, 2)$

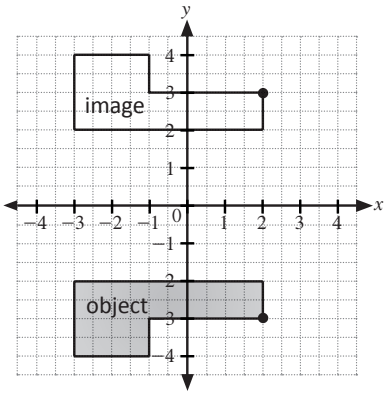
Transformation on the Cartesian number plane

3. d



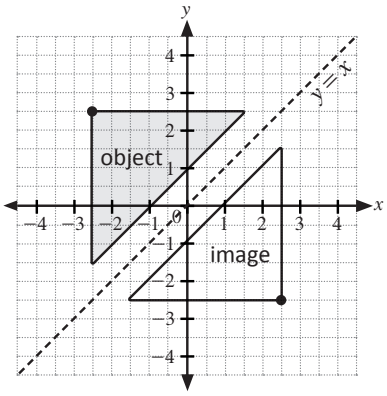
New coordinates for dot = $(2.5, 0)$

e



New coordinates for dot = $(2, 3)$

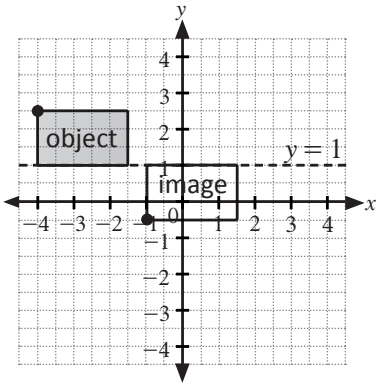
f



New coordinates for dot = $(2.5, -2.5)$

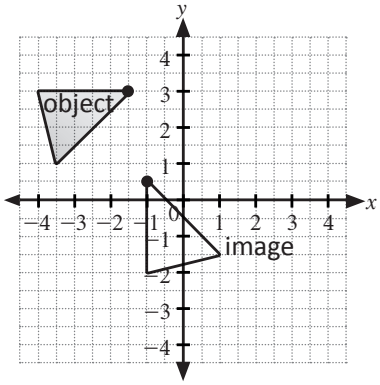
Transformation on the Cartesian number plane

4. **a**



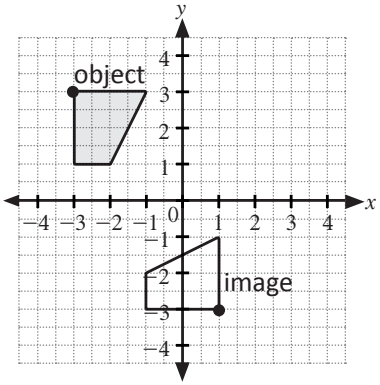
New coordinates for dot = $(-1, -1)$

b



New coordinates for dot = $(1, 0.5)$

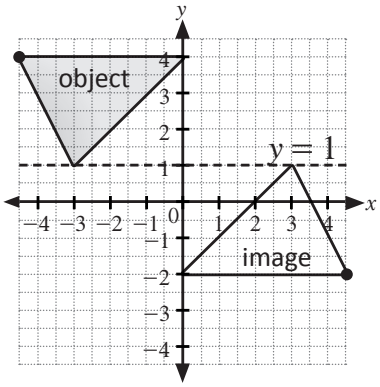
c



New coordinates for dot = $(1, -3)$

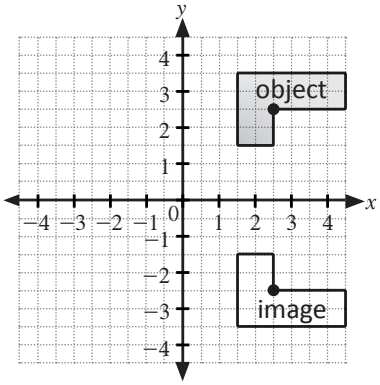
Transformation on the Cartesian number plane

4. **d**



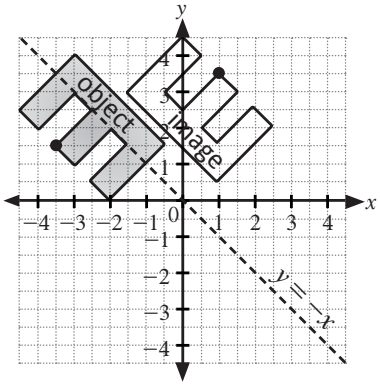
New coordinates for dot = $(4.5, -2)$

e



New coordinates for dot = $(2.5, -2.5)$

f



New coordinates for dot = $(2, 3.5)$

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Transformation on the Cartesian number plane

5.
- Reflect horizontally about the side AB. Coordinates of A: $(5, 4)$
- Translate horizontally in the negative x-direction 1 unit. Coordinates of A: $(4, 4)$
- Translate vertically in the negative y-direction 2.5 units. Coordinates of A: $(4, 1.5)$
- Translate horizontally in the negative x-direction 2 units. Coordinates of A: $(2, 1.5)$
- Reflect horizontally about the side CD. Coordinates of A: $(-2, 1.5)$
- Translate vertically in the positive y-direction 2.5 units. Coordinates of A: $(-2, 4)$
- Translate horizontally in the negative x-direction 3 units. Coordinates of A: $(-5, 4)$
- Rotate $180°$ about the centre of the player square. Coordinates of A: $(-3, 6)$
- Translate vertically in the negative y-direction 5 units. Coordinates of A: $(-3, 1)$
- Reflect vertically about the side BC. Coordinates of A: $(-3, -3)$
- Translate horizontally in the positive x-direction 1 unit. Coordinates of A: $(-2, -3)$
- Translate vertically in the negative y-direction 1 unit. Coordinates of A: $(-2, -4)$
- Translate horizontally in the positive x-direction 8 units. Coordinates of A: $(6, -4)$
- Translate vertically in the positive y-direction 1 unit. Coordinates of A: $(6, -3)$

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