Simplifying Algebra

Curriculum Ready

Mathletics

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SIMPLIFYING ALGEBRA

Algebra is mathematics with more than just numbers. Numbers have a fixed value, but algebra introduces variables – whose values can change. These are represented by letters. Although they work similarly to numbers it is important to be aware of how to add, subtract, multiply and divide expressions containing variables.

Answer these questions, *before* working through the chapter.

I used to think:

What is the difference between like and unlike terms?

Write an example explaining the distribution property

What is a binomial product?

Answer these questions, *after* working through the chapter.

But now I think:

What is the difference between like and unlike terms?

Write an example explaining the distribution property

What is a binomial product?

What do I know now that I didn't know before?

Like Terms

Terms with the same variables and indices are called **like terms**. If indices or variables differ they are called **unlike terms**.

Examples of like and unlike terms

Like Terms	**Unlike Terms**
$2p^2$ and $4p^2$	Different vairables $5m$ and $4n$
$3xy^2$ and $7xy^2$	Different indices $4a^2b$ and $4ab^2$

Only like terms can be added or subtracted.

Simplify the following as much as possible

Like terms

$$2a^2 + 3ab + 7a^2 - 2ab = 9a^2 + ab$$

Like terms

This can't be simplified anymore because these are unlike terms.

Multiplying Terms

It's easy to see that $2 \times 3 = 6$ and $3 \times 2 = 6$, so it doesn't matter in which order 2 and 3 are multiplied. This is always true! In any situation $a \times b = b \times a$. This is even true if more than two terms are multiplied: $a \times b \times c = a \times c \times b = b \times a \times c$.

This means $xy = yx$ and they are like terms. $abc = acb = bac$ are all equal and are also like terms.

The times sign $(\times)$ is dropped in multiplcation of different terms. So $4 \times x = 4x$, $a \times b = ab$ and $x \times y \times 2z = 2xyz$.

The coefficients are always multiplied separately. Here are some more examples.

Write the following products using algebra

a $m \times 3n$

$= 3mn$

b $p \times 2q \times 4r$

Coefficients multiplied separately

$= (2 \times 4) \times p \times q \times r$

$= 8pqr$

c $6xy \times 3x^2y^3$

Coefficients multiplied separately

$= (6 \times 3) \times (x \times x^2)(y \times y^3)$

$= 18x^3y^4$

Rules for Multiplication

When terms have the same sign their product is positive, when they have different signs their product is negative.

- $+ \times + = +$ ← Positive × Positive = Positive
- $+ \times - = -$ ← Positive × Negative = Negative
- $- \times + = -$ ← Negative × Positive = Negative
- $- \times - = +$ ← Negative × Negative = Positive

Here are some examples of multiplying with signs

Simplify the following

ⓐ
$$\begin{aligned}&-7a\times 2b\\ &=(-7\times 2)\times(a\times b)\\ &=-14ab\end{aligned}$$

ⓑ
$$\begin{aligned}&-3x\times -5y\\ &=(-3\times -5)\times(x\times y)\\ &=15xy\end{aligned}$$

ⓒ
$$\begin{aligned}&-2m\times 2n\times -5p\\ &=(-2\times 2\times -5)\times(m\times n\times p)\\ &=20mnp\end{aligned}$$

ⓓ
$$\begin{aligned}&-5t\times -4u\times -v\\ &=(-5\times -4\times -1)\times(t\times u\times v)\\ &=-20tuv\end{aligned}$$

The middle steps of ⓒ and ⓓ can be skipped if you remember this rule:

- If there are an **even** amount of negatives then the product is **positive**.
- If there are an **odd** amount of negatives then the product is **negative**.

Order of Operations

Remember that brackets and multiplication (and division) are always done before addition and subtraction.

Simplify the following as much as possible

ⓐ
$$3xy + \underbrace{2x\times -4y}_{\text{Multiplication FIRST}}$$
$$= 3xy - 8xy$$
Simplify like terms
$$= -5xy$$

ⓑ
$$\underbrace{-5a\times 2b}_{\text{Multiplication FIRST}} - \underbrace{6b\times -7c}_{\text{Multiplication FIRST}}$$
$$= -10ab\,\underbrace{-\,(-42ac)}_{-(-42)\,=\,42}$$
$$= -10ab + 42ac$$

1. Say if these are like or unlike terms:

a a and $2a$

b a and a^2

c pq and qp

d cd^2 and d^2c

2. Complete the following:

a $3 \times 4 = 4 \times$ ____

b $yz =$

c $ghi =$

d $abcd =$

3. Use addition and subtraction to simplify the following as much as possible:

a $4x + 5x - 3x + 1$

b $11a - 20b + 8b - 7a$

c $-2jk + 3j + 7k + 5jk + 6k$

d $6ab + 2ba - 3ab + 6ba$

e $-def + 4edf + 5de + 7ef - 2ed$

f $4x^2y + 2xy^2 - 3x^2 + 8xy^2 + 4yx^2$

4. Use multiplication and the order of operations to simplify the following as much as possible:

a $4a \times 3$

b $2x \times 6y$

c $-5t \times 5u$

d $10p \times -q$

e $7b \times 4d \times -2c$

f $-3g \times 6h \times -i$

g $-8ux \times -2v \times -3w$

h $2p \times -3q \times r \times -2s$

i $4x + 2x \times 2y$

j $3x \times -2y + 4xy$

k $5a \times 2b + 3b \times -10a$

l $-7x \times w - 4w \times -2z + wz + 2zw$

Algebraic Fractions (Dividing Terms)

When algebraic terms are divided, algebraic fractions are formed. These can be simplified by cancelling like terms. Write the division as a fraction. Always simplify the coefficients and cancel the variables where necessary.

Simplify the following as much as possible:

a $3m \div 6m$

Write as fraction

$$= \frac{3m}{6m}$$

$$= \frac{3}{6} \times \frac{m}{m}$$

m is in both the numerator and denominator, so it can be cancelled

$$= \frac{1}{2}$$

b $12x \div 4xy$

Write as fraction

$$= \frac{12x}{4xy}$$

$$= \frac{12}{4} \times \frac{x}{xy}$$

y is only in the numerator, so it is not cancelled. x is in the numerator and denominator, so x is cancelled

$$= \frac{3}{y}$$

Rules for Division

Algebraic division with signs have the same rules as multiplication: If the terms have the same sign their **quotient** is positive, when they have different signs their **quotient** is negative.

- $+ \div + = +$ or $\frac{+}{+} = +$ Positive ÷ Positive = Positive
- $+ \div - = -$ or $\frac{+}{-} = -$ Positive ÷ Negative = Negative
- $- \div + = -$ or $\frac{-}{+} = -$ Negative ÷ Positive = Negative
- $- \div - = +$ or $\frac{-}{-} = +$ Negative ÷ Negative = Positive

Remember, always write the division as a fraction first. Then simplify the coefficients and cancel the variables.

Simplify the following as much as possible:

a $15x \div -10x$

Write as fraction

$$= \frac{15x}{-10x}$$

$$= -\frac{3}{2}$$

- The coefficients have been simplified
- x is in both the numerator and denominator, so it can be cancelled
- The answer is negative $(+ \div -)$

b $-20abc \div -2ac$

Write as fraction

$$= \frac{-20abc}{-2ac}$$

$$= 10b$$

- The coefficients have been simplified
- a and c are in both the numerator and denominator, so they can be cancelled
- b cannot be cancelled because it only appears in the numerator
- The answer is positive $(- \div -)$

Order of Operations

Remember that brackets and division (and multiplication) are always done before addition and subtraction.

Simplify

a $6x - 4xy \div 2y$

$= 6x - \frac{4xy}{2y}$ ← division has been done first

$= 6x - 2x$

$= 4x$

b $36pqr \div 12r + 7p \times 4q$

$= \frac{36pqr}{12r} + 28pq$ ← multiplication and division have been done first

$= 3pq + 28pq$

$= 31pq$

Revising Index Laws

Here is a reminder of index laws.

1. Multiplication with indices: $a^m \times a^n = a^{m+n}$
2. Division with indices: $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$
3. Raising an index to an index: $(a^m)^n = a^{mn}$
4. Products raised to indices: $(ab)^m = a^m b^m$
5. Quotients raised to indices: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
6. The zero index: $a^0 = 1$ for $a \neq 0$
7. Negative indices: $a^{-n} = \frac{1}{a^n}$ or $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
8. Fractional indices: $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Index laws are used to simplify algebraic fractions. More than one law might be necessary to simplify a fraction.

Simplify

a $x^2y^3 \times x^5 \div y^7$

$= x^{2+5}y^{3-7}$

$= x^7y^{-4}$

$= x^7\left(\frac{1}{y^4}\right)$

$= \frac{x^7}{y^4}$

b $(pq^2)^3 \div (p^3q^8)^0 \times q^{-4}$

$= p^3q^6 \div 1 \times \frac{1}{q^4}$

$= \frac{p^3q^6}{q^4}$

$= p^3q^2$

c $\sqrt[4]{b^3} \times (\sqrt[4]{b})^5$

$= b^{\frac{3}{4}} \times b^{\frac{5}{4}}$

$= b^{\frac{8}{4}}$

$= b^2$

Combining Index Laws and Algebraic Fractions

The next logical step is to mix indices and algebraic fractions. Algebraic fractions with indices work with the same two steps.

- **Step 1**: Simplify the numerator and denominator separately.
- **Step 2**: Simplify the fraction and cancel variables where necessary.

Here are some examples of algebraic fraction containing indices.

Simplify as much as possible:

a

$$\frac{4y \times 2y \times 2y}{4y + 2y + 2y}$$

Simplify numerator and denominator separately

$$= \frac{16y^3}{8y}$$

$$= 2y^{3-1}$$

Simplify fraction using index laws for division

$$= 2y^2$$

b

$$\frac{(6x^2y^3z)^2}{18x^5y^2z}$$

$$= \frac{36x^4y^6z^2}{18x^5y^2z}$$

Simplify numerator and denominator separately

$$= 2x^{4-5}y^{6-2}z^{2-1}$$

Simplify fraction using index laws for division

$$= 2x^{-1}y^4z$$

$$= \frac{2y^4z}{x}$$

x is in the denominator because its index is negative

c

$$\frac{12x^6y^8}{\sqrt{16x^6y^8}}$$

$$= \frac{12x^6y^8}{(16x^6y^8)^{\frac{1}{2}}}$$

Fractional indices

$$= \frac{12x^6y^8}{4x^3y^4}$$

Simplify numerator and denominator separately

$$= 3x^{6-3}y^{8-4}$$

Simplify fraction using index laws for division

$$= 3x^3y^4$$

d

$$\frac{(2p^2q)^3 \times (4p^2q^2)^2}{-p^3q^3}$$

$$= \frac{8p^6q^3 \times 16p^4q^4}{-p^3q^3}$$

$$= \frac{128p^{10}q^7}{-p^3q^3}$$

Simplify numerator and denominator separately

$$= -128p^{10-3}q^{7-3}$$

Simplify fraction using index laws for division

$$= -128p^7q^4$$

1. Find the quotient of the following:

a $8a \div 2a$

b $a \div a^2$

c $-4pqr \div 2pr$

d $-100amn \div -20n$

e $18wx \div -9xy$

f $-5st \div 20stu$

2. Answer these questions about order of operations:

a Simplify $4a \times 2b$.

b Simplify $2b \times 4a$.

c Is this statement true or false: $4a \times 2b = 2b \times 4a$.

d Is the order of the terms in multiplication important?

e Simplify $4a \div 2b$.

f Simplify $2b \div 4a$.

g Is this statement true or false: $4a \div 2b = 2b \div 4a$.

h Is the order of the terms in division important?

3. Use the order of operations to simplify these as much as possible:

a $10a - 40ab \div 10b$

b $26xyz \div 2xz + 6y$

c $14mn \div 2n - 6m$

d $xz \times 24xyz \div 4xz$

e $20xy \div 5x + 30yz \div 6z$

f $15d \times 3f - 200def \div 10e$

4. Simplify these algebraic fractions as much as possible:

a $\dfrac{3y + 4y + 5y}{2y \times 3y \times y}$

b $\dfrac{(4x^3)^2}{(2x^2)^4 \times 2x}$

c $\dfrac{(5xy^2)^2 \times (2x^2y)^3}{(2x^2y^2)^3}$

d $\dfrac{(3pq)^2 \times (-p^3q^3)}{-pq^2}$

e $\dfrac{9f^2g^3 \times (2fg)^3}{(3f^2g)^2 \times 2f^4g}$

f $\left(\dfrac{-qr^2 \times q^3r}{(3q^2r)^2}\right)^2$

5. Answer these questions:

a Use the index law $a^{-n} = \frac{1}{a^n}$ to prove $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$. (Hint: $\left(\frac{a}{b}\right)^{-n} = \left(\left(\frac{a}{b}\right)^{-1}\right)^{n}$)

b Use this result to simplify this fraction $\left(\frac{(3xy^2)^2}{(3x^2y)^3}\right)^{-2}$.

Expanding Brackets

A term outside a bracket is multipled with all terms inside a bracket, so that:

$$a(b+c) = a\times b + a\times c = ab + ac$$

This property of multiplication with brackets is called the ‘distribution property’. Here are some examples:

Expand the following

a $3(2+x)$

$\boxed{3\times 2}$ $\boxed{3\times x}$

$= 6 + 3x$

b $x(5+2x+3y)$

$\boxed{x\times 5}$ $\boxed{x\times 2x}$ $\boxed{x\times 3y}$

$= 5x + 2x^2 + 3xy$

There is no difference if the number on the outside is negative. Always multiply the term outside with **all the terms** inside.

Expand the following

a $-4(y-5)$

$\boxed{-4\times y}$ $\boxed{-4\times -5}$

$= -4y + 20$

b $-3p(2q+6p-9)$

$\boxed{-3p\times 2q}$ $\boxed{-3p\times 6p}$ $\boxed{-3p\times -9}$

$= -6pq - 18p^2 + 27p$

Sometimes like terms can be simplified after expanding brackets.

Expand the following

a $4(2x-x^2) + 3x(5-2x)$

Like terms

$= 8x - 4x^2 + 15x - 6x^2$

Like terms

$= 23x - 10x^2$

b $-2q(1+4p-3pq) + q(2p-5pq)$

Like terms

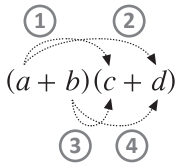
$= -2q - 8pq + 6pq^2 + 2pq - 10pq^2$

Like terms

$= -2q - 6pq - 4pq^2$

Multiplying Brackets

Brackets can also be multiplied together. Both terms in the first bracket are multiplied with both terms in the second bracket.



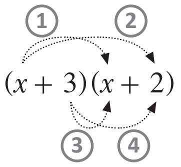
$$= \overset{①}{ac} + \overset{②}{ad} + \overset{③}{bc} + \overset{④}{bd}$$

The product of two brackets can also be thought of in this way:

$$\begin{aligned}(a+b)(c+d) &= a(c+d) + b(c+d)\\ &= ac + ad + bc + bd\end{aligned}$$

Find the following products:

a

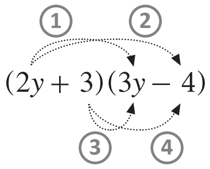


$$= \overset{①}{(x \times x)} + \overset{②}{(x + 2)} + \overset{③}{(3 \times x)} + \overset{④}{(3 \times 2)}$$

$$= x^2 + \underbrace{2x + 3x}_{\text{Like terms}} + 6$$

$$= x^2 + 5x + 6$$

b



$$= \overset{①}{(2y \times 3y)} + \overset{②}{(2y \times -4)} + \overset{③}{(3 \times 3y)} + \overset{④}{(3 \times -4)}$$

$$= 6y^2 - \underbrace{8y + 9y}_{\text{Like terms}} - 12$$

$$= 6y^2 + y - 12$$

c $(2p + 3q)(p - 4q)$

$$= (2p \times p) + (2p \times -4q) + (3q \times p) + (3q \times -4q)$$

$$= 2p^2 - 8pq + 3pq - 12q^2$$

$$= 2p^2 - 5pq - 12q^2$$

d $(3a - 5b)(3c + 2d)$

$$= (3a \times 3c) + (3a \times 2d) + (-5b \times 3c) + (-5b \times 2d)$$

$$= 9ac + 6ad - 15bc - 10bd$$

1. Expand these brackets:

a $5(2y+3)$

b $6(4-3t)$

c $-4(-3-5m)$

d $-3(9-4x)$

e $\frac{1}{2}(8x-2y)$

f $-\frac{1}{4}(16n-24m)$

g $2x(4x-2y)$

h $x^2(4xy-3y)$

i $5p(q+2pq+3p)$

j $3xy(2x-2y+1)$

2. Expand these brackets:

a $x(2+3x-4y+4xy)$

b $-2abc(3-ab-4ac+2a^2b-5bc+3c^2)$

3. Expand then simplify using like terms:

a $4(x-3x^2)+2x(7x-5)$

b $d^2(1-2c+3cd)+cd(5d-3cd^2-2)$

c $5p(3q-2pq^2)-4q(2p^2q+2p-3)$

d $-3ab(a^2b-2b^2+6a)-b(8a^3b+2ab^2-4a^2)+a^2b$

4. Expand and simplify:

a $(2x + 2)(x - 1)$

b $(2st + 3t)(4t - 2s)$

c $(4k - 1)(3 - 2k)$

d $(ab - 2a)(3b^2 + 6ab)$

e $(x + y)(x - y)$

f $(ab - cd)(ab + cd)$

g $(3x^2y - 4xy^2)(xy + 2x)$

h $(3ab^3c - 4a^2bc^2)(5abc - 2a^2b^2c^2)$

Adding and Subtracting Algebraic Fractions

Like all fractions, algebriac fractions can only be added or subtracted if they have the same denominator.
If the fractions already have a common denominator, then simplify the numerators only.

Write all of these as a single algebraic fraction

a $\frac{9h}{19} - \frac{4h}{19}$

$= \frac{9h - 4h}{19}$ ← These fractions already have a common denominator

$= \frac{5h}{19}$

b $\frac{y}{3} + \frac{3y}{4} - \frac{y}{6}$ ← 12 is the lowest common denominator of 3, 4 and 6

$= \frac{4y + 9y - 2y}{12}$

$= \frac{11y}{12}$

c $\frac{5}{ab} + \frac{5}{a}$ ← ab is the lowest common denominator of a and ab

$= \frac{5 + 5b}{ab}$

d $\frac{8p}{15} - \frac{2q}{30} + \frac{2p + q}{10}$ ← 30 is the lowest common denominator of 15, 30 and 10

$= \frac{16p - 2q + 6p + 3q}{30}$

$= \frac{25q}{30} = \frac{5q}{6}$

Sometimes its difficult to find a common denominator. In these situations, the common denominator can be found by multiplying the denominators together.

Adding and subtracting fractions with complicated denominators

a $\frac{2}{x + 3} + \frac{5}{7}$

***Step* 1:** Find a common denominator

$7(x + 3)$ ← This is the product of the original denominators

***Step* 2:** Write both fractions over this denominator

$= \frac{14}{7(x + 3)} + \frac{5(x + 3)}{7(x + 3)}$

$= \frac{14 + 5(x + 3)}{7(x + 3)}$

***Step* 3:** Simplify

$= \frac{14 + 5x + 15}{7x + 21}$ ← Use the distributive property to expand the bracket

$= \frac{5x + 29}{7x + 21}$

b $\frac{2}{m + n} - \frac{2}{m - n}$

***Step* 1:** Find a common denominator

$(m + n)(m - n)$ ← This is the product of the original denominators

***Step* 2:** Write both fractions over this denominator

$= \frac{2(m - n)}{(m + n)(m - n)} - \frac{2(m + n)}{(m + n)(m - n)}$

$= \frac{2(m - n) - 2(m + n)}{(m + n)(m - n)}$

***Step* 3:** Simplify

$= \frac{2m - 2n - 2m - 2n}{m^2 + mn - mn - n^2}$ ← Multiply the brackets to find this denominator

$= -\frac{4n}{m^2 - n^2}$

5. Simplify these fractions:

a $\frac{7x}{4}+\frac{5x}{4}$

b $\frac{2x}{3}-\frac{5x}{12}$

c $\frac{3}{mn}+\frac{2}{km}$

d $\frac{5}{6}+\frac{2}{1-2x}$

e $\frac{3}{2a-1}-\frac{4}{a}$

f $\frac{3}{3y+1}+\frac{2}{5y}$

g $\frac{4}{y+6}-\frac{1}{y-1}$

h $\frac{3}{p}+\frac{5}{2p}-\frac{2}{p^2}$

Multiplying and Dividing Algebraic Fractions

Multiplying and dividing algebraic fractions is easy because a common denominator is not necessary.
To multiply algebraic fractions, simply multiply the numerators and the denominators separately.

Find these products:

a $\frac{3}{x} \times \frac{y}{4}$

$= \frac{3 \times y}{x \times 4}$ ← Multiply numerators and denominators separately

$= \frac{3y}{4x}$

b $\frac{2p}{3} \times \frac{p}{q}$

$= \frac{2p \times p}{3 \times q}$ ← Multiply numerators and denominators separately

$= \frac{2p^2}{3q}$

c $\frac{t^4}{5} \times \frac{10}{t^2}$

$= \frac{t^4 \times 10}{5 \times t^2}$ ← Multiply numerators and denominators separately

$= \frac{10t^4}{5t^2}$

$= 2t^{4-2}$

$= 2t^2$

d $\frac{a^7 \times b^2}{2} \times \frac{24}{2a^4b^4}$

$= \frac{a^7b^2 \times 24}{2 \times 2a^4b^4}$ ← Multiply numerators and denominators separately

$= \frac{24a^7b^2}{4a^4b^4}$

$= 6a^{7-4}b^{2-4}$

$= \frac{6a^3}{b^2}$

To divide algebraic fractions, simply find the reciprocal of the second fraction and multiply.

Find these quotients:

a $\frac{2x}{8} \div \frac{y}{16}$

$= \frac{2x}{8} \times \frac{16}{y}$ ← 'flip' the second fraction to find the reciprocal.

$= \frac{2x \times 16}{8 \times y}$

$= \frac{32x}{8y}$

$= \frac{4x}{y}$

b $\frac{-3ab}{4c} \div \frac{12b^2}{24c}$

$= \frac{-3ab}{5c} \times \frac{24c}{12b^2}$ ← 'flip' the second fraction to find the reciprocal.

$= \frac{-3ab \times 24c}{5c \times 12b^2}$

$= \frac{-72abc}{60b^2c}$

$= -\frac{6a}{5b}$

6. Write these in simplest form:

a $\frac{p}{2} \times \frac{p}{10}$

b $\frac{mn}{5} \times \frac{10}{n}$

c $\frac{mn}{5} \times \frac{25}{m^2 n^3}$

d $\frac{3p^2 q}{7} \times \frac{6q}{p^4}$

e $\frac{8s^3 t^2}{9p} \times \frac{27p^3 s^2}{16t^4}$

f $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d}$

g $\frac{3d}{5e^3 f^2} \times \frac{4e^4 f^5}{3d^4} \times \frac{d^2 e f^3}{4}$

h $\frac{-5wx^2 y^3}{2a^2} \times \frac{10a^4 b}{25wxy}$

7. Write these in simplest form:

a $\frac{3p}{2} \div \frac{q}{6}$

b $\frac{a^2}{b} \times \frac{a}{2}$

c $\frac{4a^3b^5}{3} \times \frac{2a^2b^2}{9}$

d $\frac{3d^2e}{8f^2} \div \frac{9e^3}{16d^2f}$

e $\frac{5s^3t^2}{x^3y^4} \div \frac{20s^5t}{x^6y^2}$

f $\frac{6a^2b^3c^2}{7p^4q^3r^2} \div \frac{2a^3bc^4}{21p^2q^5r}$

Shortcuts for Multiplying Brackets

A product of two brackets it called a ‘binomial product’. Some binomial products can be found more easily than others. Look at this example:

$$\begin{aligned}&(a+b)(a-b)\\ &= a^2 \underbrace{-\,ab + ab}_{\boxed{\text{The middle terms cancel each other away}}} - b^2\\ &= a^2 - b^2\end{aligned}$$

Can you see the shortcut? Their product is the square of the first term minus the square of the second term. Here are some examples using this shortcut:

Find these products:

a

$$\begin{aligned}&(x+2)(x-2)\\ &= \underset{\boxed{\substack{\text{1st term}\\ \text{squared}}}}{\underset{\uparrow}{(x)^2}} - \underset{\boxed{\substack{\text{2nd term}\\ \text{squared}}}}{\underset{\uparrow}{(2)^2}}\\ &= x^2 - 4\end{aligned}$$

b

$$\begin{aligned}&(3y-5)(3y+5)\\ &= \underset{\boxed{\substack{\text{1st term}\\ \text{squared}}}}{\underset{\uparrow}{(3y)^2}} - \underset{\boxed{\substack{\text{2nd term}\\ \text{squared}}}}{\underset{\uparrow}{(5)^2}}\\ &= 9y^2 - 25\end{aligned}$$

What about the perfect square of a bracket? Look at this example:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2 + ab + ab + b^2\\ &= \underset{\boxed{\substack{\text{1st term}\\ \text{squared}}}}{\underset{\uparrow}{a^2}} + \underset{\boxed{\substack{2\times\text{ product}\\ \text{of terms}}}}{\underset{\uparrow}{2ab}} + \underset{\boxed{\substack{\text{2nd term}\\ \text{squared}}}}{\underset{\uparrow}{b^2}}\end{aligned}$$

Can you see the shortcut? Here are some examples using this shortcut:

Find these products:

a

$$\begin{aligned}&(x+4)^2\\ &= \underset{\boxed{\substack{\text{1st term}\\ \text{squared}}}}{\underset{\uparrow}{(x^2)}} + \underset{\boxed{\substack{2\times\text{ product}\\ \text{of terms}}}}{\underset{\uparrow}{2(x)(4)}} + \underset{\boxed{\substack{\text{2nd term}\\ \text{squared}}}}{\underset{\uparrow}{(4)^2}}\\ &= x^2 + 8x + 16\end{aligned}$$

b

$$\begin{aligned}&(4p-2q)^2\\ &= \underset{\boxed{\substack{\text{1st term}\\ \text{squared}}}}{\underset{\uparrow}{(4p)^2}} + \underset{\boxed{\substack{2\times\text{ product}\\ \text{of terms}}}}{\underset{\uparrow}{2(4p)(-2q)}} + \underset{\boxed{\substack{\text{2nd term}\\ \text{squared}}}}{\underset{\uparrow}{(-2q)^2}}\\ &= 16p^2 - 16pq + 4q^2\end{aligned}$$

These shortcuts may seem silly for these easier examples, but they are especially useful for more complicated examples.

Find these products:

a $\left(\frac{4}{x}+x\right)\left(\frac{4}{x}-x\right)$

$= \left(\frac{4}{x}\right)^2-(x)^2$

$= \frac{16}{x^2}-x^2$

b $(3x^2y+4pq^3)(3x^2y-4pq^3)$

$= (3x^2y)^2-(4pq^3)^2$

$= 9x^4y^2-16p^2q^6$

c $\left(5y-\frac{3}{2y}\right)^2$

$= (5y)^2+2(5y)\left(-\frac{3}{2y}\right)+\left(-\frac{3}{2y}\right)^2$

$= 25y^2+2\left(-\frac{15y}{2y}\right)+\frac{9}{4y^2}$

$= 25y^2-15+\frac{9}{4y^2}$

d $(2m^3n^2-3n)^2$

$= (2m^3n^2)^2+2(2m^3n^2)(-3n)+(-3n)^2$

$= 4m^6n^4+2(-6m^3n^3)+9n^2$

$= 4m^6n^4-12m^3n^3+9n^2$

We can even use these rules in calculations using actual numbers.

a 11^2-9^2

$= (11+9)(11-9)$

$= (20)(2)$

$= 40$

b 36^2

$= (30+6)^2$

$= 30^2+2(30)(6)+6^2$

$= 900+360+36$

$= 1296$

1. Prove that $(x+y)(x-y)=x^2-y^2$**.**

---

2. Simplify these perfect squares:

a $(2p+1)^2$

b $(5x-7y)^2$

c $(-3m-2n)^2$

d $(3p-pq)^2$

e $\left(\frac{1}{x}+x\right)^2$

f $\left(2y^2-\frac{3}{y}\right)^2$

3. Simplify these binomial products:

a $(x+3)(x-3)$

b $(5-m)(5+m)$

c $(2b+3c)(2b-3c)$

d $(7x+3y)(7x-3y)$

e $(xy+x^2y^2)(xy-x^2y^2)$

f $(10a^2b^3-4ab^2)(10a^2b^3+4ab^2)$

g $\left(\frac{1}{2m}+m^2\right)\left(\frac{1}{2m}-m^2\right)$

h $\left(\frac{2}{5x}-3y\right)\left(\frac{2}{5x}+3y\right)$

4. Find △ and □ in each of the following:

a $(x + 3)^2 = x^2 + \triangle x + 9$

b $(2x + \triangle)^2 = 4x^2 + 12xy + 9y^2$

c $(\triangle - 5q)^2 = 16p^2 - 40pq + 25q^2$

d $(2m + \triangle)^2 = 4m^2 + 16mn^2 + \square$

5. Find these values using expanded form:

a $16^2 - 6^2$

b $64^2 - 36^2$

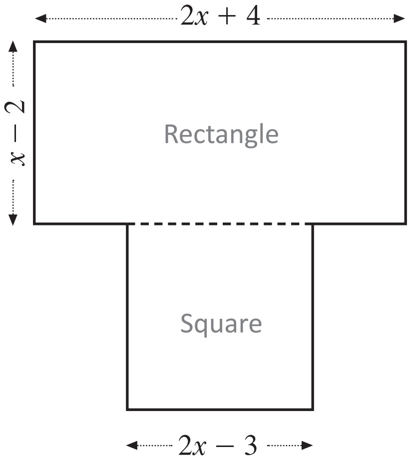
c 29^2

d 37^2

e $77^2 - 27^2$

f 12.6^2 (Hint: $12.6 = 12 + 0.6$)

6. A stage in a stadium is made up of a rectangle and a square in the following diagram. The dimensions are in terms of x.



a Find expression for the area of the square in the form of a binomial product.

b Find expression for the area of the rectangle in the form of a binomial product.

c Show that the total area of the stage is $(x - 2)(2x + 4) + (2x - 3)^2$.

d Simplify the expression for the area of the stage.

e Find the value of the area of the stage if $x = 5$.

Basics:

1. a Like terms
   b Unlike terms
   c Like terms
   d Like terms

2. a $3 \times 4 = 4 \times 3$
   b $yz = z \times y$
   c $ghi = g \times h \times i$
   d $abcd = a \times b \times c \times d$

3. a $6x + 1$
   b $4a - 12b$
   c $3j + 13k + 3jk$
   d $11ab$
   e $3edf + 3ed + 7ef$
   f $8x^2y + 10xy^2 - 3x^2$

4. a $12a$
   b $12xy$
   c $-25tu$
   d $-10pq$
   e $-56bcd$
   f $18ghi$
   g $-48uvwx$
   h $12pqrs$
   i $4x + 4xy$
   j $-2xy$
   k $-20ab$
   l $11zw - 7xw$

Knowing More:

1. a 4
   b $\frac{1}{a}$
   c $-2q$
   d $5am$
   e $-\frac{2w}{y}$
   f $-\frac{1}{4u}$

2. a $8ab$
   b $8ab$
   c True, they are both $8ab$.
   d No, the answers are the same.

Knowing More:

2. e $\frac{2a}{b}$
   f $\frac{b}{2a}$
   g False, they are not equal.
   h Yes, the answers are different.

3. a $6a$
   b $19y$
   c m
   d $6xyz$
   e $9y$
   f $25df$

4. a $\frac{2}{y^2}$
   b $\frac{1}{2x^3}$
   c $25x^2y$
   d $9p^4q^3$
   e $\frac{4g^3}{f^3}$
   f $\frac{r^2}{81}$

5. a $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$
   b $\frac{9x^8}{y^2}$

Using Our Knowledge:

1. a $10y + 15$
   b $24 - 18t$
   c $12 + 20m$
   d $27 + 12x$
   e $4x - y$
   f $-4n + 6m$
   g $8x^2 - 4xy$
   h $4x^3y - 3yx^2$
   i $5pq + 10p^2q + 15p^2$
   j $6x^2y - 6xy^2 + 3xy$

2. a $2x + 3x^2 - 4xy + 4x^2y$
   b $-6abc+2a^2b^2c+8a^2bc^2-4a^3b^2c+10ab^2c^2-6abc^3$

Using Our Knowledge:

3. a $2x^2 - 6x$

b $3cd^3 + 3cd^2 + d^2 - 3c^2d^3 - 2cd$

c $7pq + 12q - 18p^2q^2$

d $4ab^3 - 13a^2b - 11a^3b^2$

4. a $2x^2 - 2$

b $8st^2 - 4s^2t + 12t^2 - 6st$

c $-8k^2 + 14k - 3$

d $3ab^3 + 6a^2b^2 - 6ab^2 - 12a^2b$

e $x^2 - y^2$

f $a^2b^2 - c^2d^2$

g $3x^3y^2 + 6x^3y - 4x^2y^3 - 8x^2y^2$

h $15a^2b^4c^2 - 6a^3b^5c^3 - 20a^3b^3c^3 + 8a^4b^3c^4$

5. a $3x$ b $\frac{x}{4}$

c $\frac{3k + 2n}{kmn}$ d $\frac{17 - 10x}{6 - 12x}$

e $\frac{-5a + 4}{a(2a - 1)}$ f $\frac{21y + 2}{5y(3y + 1)}$

g $\frac{3y - 10}{(y - 1)(y + 6)}$ h $\frac{11p - 4}{2p^2}$

6. a $\frac{p^2}{20}$ b $2m$

c $\frac{5}{mn^2}$ d $\frac{18q^2}{7p^2}$

e $\frac{3p^2s^5}{2t^2}$ f $\frac{a}{d}$

g $\frac{e^2f^6}{5d}$ h $-a^2bxy^2$

Using Our Knowledge:

7. a $\frac{9p}{q}$ b $\frac{a^3}{2b}$

c $\frac{8a^5b^7}{21}$ d $\frac{2d^4}{3fe^2}$

e $\frac{tx^3}{4s^2y^2}$ f $\frac{9b^2q^2}{ac^2p^2r}$

Thinking More:

2. a $4p^2 + 4p + 1$

b $25x^2 - 70xy + 49y^2$

c $9m^2 + 12mn + 4n^2$

d $9p^2 - 6p^2q + p^2q^2$

e $\frac{1}{x^2} + x^2 + 2$

f $4y^4 - 12y + \frac{9}{y^2}$

3. a $x^2 - 9$ b $25 - m^2$

c $4b^2 - 9c^2$ d $49x^2 - 9y^2$

e $x^2y^2 - x^4y^4$

f $100a^4b^6 - 16a^2b^4$

g $\frac{1}{4m^2} - m^4$

h $\frac{4}{25x^2} - 9y^2$

Thinking More:

4. (a) $\triangle = 6$ (b) $\triangle = 3y$

(c) $\triangle = 4p$ (d) $\triangle = 4n^2$

$\square = 16n^4$

5. (a) 220 (b) 2800

(c) 841 (d) 1369

(e) 5200 (f) 158.76

6. (a) $(2x - 3)^2$

(b) $(x - 2)(2x + 4)$

(d) $6x^2 - 12x + 1$

(e) 91

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